Day 83 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Conditional probability from given values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, P(A) = 0.9, P(B) = 0.8 and P(A | B) = 0.9.
(a) Find P(A ∩ B).
(b) Find P(A′ | B′).

P(A ∩ B) = P(A | B) × P(B) = 0.9 × 0.8 = 0.72
P(A ∪ B) = 0.9 + 0.8 − 0.72 = 0.98
P(B′) = 0.2
P(A′ ∩ B′) = 1 − P(A ∪ B) = 0.02
$P(A' \mid B') = \frac{0.02}{0.2}$

Answer: (a) 0.72 (b) $\frac{1}{10}$

2 Equations quadratic in an exponential (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2^{2x} - 13 \times 2^x + 40 = 0$,
giving each answer exactly and, where not an integer, to 3 s.f.

Let $u = 2^x$: $u^2 - 13u + 40 = 0$
$(u - 8)(u - 5) = 0$, so $2^x = 8$ or $2^x = 5$
Take logarithms: $x = \log_2 5 = 2.32$; $x = \log_2 8 = 3$
Answer: $x = \log_2 5 = 2.32$; $x = \log_2 8 = 3$

3 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The daily output of a machine is N(μ, 25) with known variance 25. The mean was claimed to be 61 units.
A random sample of 36 has mean 59.75. Test at the 5% level whether the mean is greater than 61.

H_0: μ = 61, H_1: μ > 61. Under H_0, the sample mean ~ $N(61, \frac{25}{36})$
$z = \frac{59.75 - 61}{5 \div \sqrt{36}} = -1.500$
p-value = P(Z > −1.500) = 0.933
0.933 > 5%, so do not reject H_0
There is insufficient evidence at the 5% level that the mean has changed as claimed
Answer: Do not reject H_0 (z = −1.500, p-value 0.933)

4 Overtaking with a delayed start (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car P passes a point O travelling at a constant speed of 24.3 m s^{-1}. 2 s later, car Q leaves O from rest with constant acceleration 3 m s^{-2}, travelling in the same direction.
Q overtakes P at the point X. Find (a) the time Q takes to reach X, (b) the distance OX, (c) the speed of Q at X.

Let Q travel for t s. P: distance = 24.3(t + 2); Q: distance = ½ × 3 × t^2
Equate: $3t^2 - 48.6t - 97.2 = 0$, so $(t - 18)(3t + 5.4) = 0$
t = 18 (the other root is negative)
OX = $3 \times 18^2 / 2 = 486$ m, and speed = 3 × 18 = 54
Answer: (a) 18 s (b) 486 m (c) 54 m s^{-1}

5 Exponential times trigonometric (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = e^{3x} \cos 3x$. Show that $dy/dx = e^{3x}(3 \cos 3x - 3 \sin 3x)$.
Find the x coordinate of the stationary point with the smallest positive x (3 s.f.), and its nature.

Product rule: $dy/dx = (3)e^{3x} \cos 3x - e^{3x} \times 3 \sin 3x$
dy/dx = 0 requires 3 cos 3x = 3 sin 3x
tan 3x = 1, smallest positive 3x = 0.7854
x = 0.262
Second derivative test (or gradient change): a maximum
Answer: x = 0.262, maximum

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6 Transformations: new equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 2x^2 + 2x + 5$ is translated by $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$
and then a stretch, scale factor 2, parallel to the y-axis.
Find the equation of the resulting curve in the form $y = ax^2 + bx + c$.

Translation by $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$: $y = f(x - 1) - 2 = 2x^2 - 2x + 3$.

Then a stretch, scale factor 2, parallel to the y-axis: multiply y by 2.
Expand and simplify to give the quadratic.

Answer: $y = 4x^2 - 4x + 6$

7 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^3\sqrt{112 - x}$, for $0 < x < 112$, has one stationary point. Find its exact coordinates and determine its nature.

$\mathrm{d}y/\mathrm{d}x = \dfrac{x^2(672 - 7x)}{2\sqrt{112 - x}}$

$\mathrm{d}y/\mathrm{d}x = 0$: $672 - 7x = 0$, so $x = 96$

$y = 96^3 \times 4$

Gradient changes from positive to negative: a maximum

Answer: (96, 3538944), maximum

8 Periodic recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_1 = 5$ and

$u_{n+1} = \dfrac{1}{1 - u_n}$.

Show that the sequence is periodic and find

$\sum_{r=1}^{99} u_r$.

$u_1 = 5$, $u_2 = -\dfrac{1}{4}$, $u_3 = \dfrac{4}{5}$, $u_4 = 5$.

$u_4 = u_1$, so the sequence repeats with period 3.

One period sums to $\dfrac{111}{20}$.

$99 = 3 \times 33 + 0$: 33 full periods plus the first 0 terms.

Answer: $\dfrac{3663}{20}$

9 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A shell is projected from level ground with horizontal and vertical velocity components 34.3 and 53.9 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take $g = 9.8$ m s^{-2}.

At 45° to the horizontal, $|v_y| = v_x = 34.3$

Moving upwards: $53.9 - 9.8t = 34.3$, so $t = 2$

Moving downwards: $53.9 - 9.8t = -34.3$, so $t = 9$

Horizontal distance $= 34.3 \times (9 - 2)$

Answer: (a) 2 s and 9 s (b) 240.1 m

10 Parametric curve and line intersection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has parametric equations
$x = 2t$, $y = 4t^2$. The line $y = 8x - 12$
meets C at A and B. Find the exact length AB.

Substitute: $4t^2 = 16t - 12$.
$4t^2 - 16t + 12 = 0 \Rightarrow t = 1$ or $t = 3$.
A(2, 4), B(6, 36).
$AB^2 = (-4)^2 + (-32)^2 = 1040$.
Answer: AB = $4\sqrt{65}$