Day 81 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Large data set: linear change of units (AQA, Edexcel, OCR, MEI, Eduqas)

Large Data Set, daily mean wind speed (knots), Leeming, August 2015 (n = 92): mean 9.9, sd 4.5. (1 knot = 1.852 km/h)
(a) Find the mean and sd in km/h.
(b) Is a reading of 17.6 more than 2 sd above the mean?

New value = 1.852 × old, so mean = 1.852 × 9.9 = 18.33
The constant shifts the mean but not the spread; only the multiplier affects σ
σ = 1.852 × 4.5 = 8.334
Outlier cut-off = mean + 2σ = 9.9 + 9 = 18.9 (original units)
17.6 is less than 18.9, so no
Answer: (a) mean 18.3, sd 8.33 (b) no

2 Arithmetic series modelling (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first row of a theatre has 19 seats. Each row has 5 more seats than the row in front. Find
(a) the number of seats in row 29,
(b) the least number of rows for at least 1691 seats.

Arithmetic: a = 19, d = 5.
(a) $u_{29} = 19 + 28 \times 5 = 159$.
(b) $S_n = \frac{n}{2}(38 + 5n - 5) = \frac{n(5n + 33)}{2} \geq 1691$.
$S_{22} = 1573 < 1691$, $S_{23} = 1702 \geq 1691$.
Answer: (a) 159 (b) 23 rows

3 Exponential growth and decay models (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population of bacteria is modelled by $P = P_0 e^{kt}$, t in hours.
Initially P = 250 and after 6 hours P = 750.
Find k, and the time for the population to reach 2000, to 3 s.f.

$P_0 = 250$. $250\, e^{6k} = 750$ so $e^{6k} = 3$
$k = \frac{1}{6} \ln 3 = 0.183$
$250\, e^{kt} = 2000$ so $e^{kt} = 8$, $t = \frac{1}{k} \ln 8$
t = 11.4 hours
Answer: k = 0.183, t = 11.4 hours

4 Particle in equilibrium on two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P of weight 78 N hangs in equilibrium from two light strings AP and BP attached to a horizontal ceiling at A and B (see diagram), where $\tan\alpha = \frac{12}{5}$ and $\tan\beta = \frac{5}{12}$.
Find (a) the tension in each string, (b) the greatest weight P could have if neither tension may exceed 132 N.

Horizontally: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_1 \times \frac{5}{13} = T_2 \times \frac{12}{13}$
Vertically: $T_1 \sin\alpha + T_2 \sin\beta = 78$, and $T_2 = \frac{5}{12} T_1$
$T_1(\frac{12}{13} + \frac{5}{12} \times \frac{5}{13}) = 78$, so $T_1 = 72$ and $T_2 = 30$
The tension in AP is the larger: it is 72 when W = 78
Tensions are proportional to W, so $W_{max} = 78 \times \frac{132}{72} = 143$
Answer: (a) AP: 72 N, BP: 30 N (b) 143 N

5 Tangent and axes triangle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tangent to $y = x\sqrt{x+2}$ at the point where $x = 2$ meets the axes at A and B. Find the exact area of triangle OAB, where O is the origin.

At $x = 2$: $y = 4$. Differentiate: gradient $= \frac{5}{2}$

Tangent: $y - 4 = \frac{5}{2}(x - 2)$, so $y = \frac{5}{2}x - 1$

y-intercept -1; x-intercept $\frac{2}{5}$

Area $= \frac{1}{2} \times |-1| \times |\frac{2}{5}|$

Answer: $\frac{1}{5}$

6 Least number of trials (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each component is faulty with probability 0.2, independently.
Find the least number n of components that must be tested so that the probability of finding at least one faulty is greater than 0.99.

P(at least one faulty) $= 1 - 0.8^n > 0.99$
$0.8^n < 0.01$
$n \ln 0.8 < \ln 0.01$
Dividing by ln 0.8 (negative) reverses the inequality: $n > 20.64$
$n = 21$
Answer: n = 21

7 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x^3 + 7x^2 - 15x + 9 \leq 0$.

Use the factor theorem to find roots: 1, 3.
$f(x) = -(x - 1)(x - 3)^2$.
Sign of f in each region (left to right): +, −, −.
Take the regions where $f \leq 0$ (include roots if ≤ or ≥).
Answer: $x \geq 1$

8 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^{\pi/2} (x + 3) \sin x \, dx$

Let $u = x + 3$ and $dv/dx = \sin x$
Integral $= [-\frac{(x+3)\cos x}{1}] + \int \frac{\cos x}{1} \, dx$

Answer: 4

9 Particle on a rough table, string slack when B lands (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (5 kg) and B (5 kg) are joined by a light string over a smooth pulley (see diagram). A lies on a rough table with $\mu = 0.4$, and B is 3 m above the floor.
The system is released from rest. Find the total distance A moves before coming to rest (A does not reach the pulley).

Until B lands, A: $T - 0.4 \times 5 \times 9.8 = 5a$. B: $5 \times 9.8 - T = 5a$
Adding: $(5 - 0.4 \times 5) \times 9.8 = 10a$, so $a = 2.94$
$v^2 = 2 \times 2.94 \times 3 = 17.64$, so $v = 4.2$

After B lands, A decelerates at $\mu g = 3.92$: extra distance $= \frac{4.2^2}{2 \times 3.92} = 2.25$

Total distance $= 3 + 2.25$
Answer: 5.25 m

10 Line tangent to a hyperbola (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = \frac{x - 5}{x - 1}$.

(a) Write down the equations of the asymptotes of C.
(b) The line $y = x + k$ is a tangent to C.
Find the possible values of k.

(a) Vertical: $x = 1$. As $x \to \infty$, $y \to 1$, so $y = 1$.
(b) $x - 5 = (x + k)(x - 1)$.
$x^2 + (k - 2)x + (-k + 5) = 0$.
Tangent: $b^2 - 4ac = 0$, which simplifies to $k^2 - 16 = 0$.
Answer: (a) $x = 1$, $y = 1$ (b) $k = -4$ or $k = 4$