Day 79 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Simultaneous logarithmic equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
$\log_5 x + \log_5 y = 5$
$x - y = 100$

$\log_5(xy) = 5$, so $xy = 5^5 = 3125$

$x = y + 100$: $y^2 + 100y - 3125 = 0$
$(y - 25)(y + 125) = 0$; $y > 0$ so $y = 25$
Then $x = y + 100$
Answer: x = 125, y = 25

2 Large data set: linear change of units (AQA, Edexcel, OCR, MEI, Eduqas)

Large Data Set, daily mean air temperature (°C), Heathrow, July 2015 (n = 184): mean 11.6, sd 3.8. (F = 1.8C + 32)
(a) Find the mean and sd in °F.
(b) Is a reading of 17.3 more than 2 sd above the mean?

New value = 1.8 × old + 32, so mean = 1.8 × 11.6 + 32 = 52.88
The constant shifts the mean but not the spread; only the multiplier affects σ
σ = 1.8 × 3.8 = 6.840
Outlier cut-off = mean + 2σ = 11.6 + 7.6 = 19.2 (original units)
17.3 is less than 19.2, so no
Answer: (a) mean 52.9, sd 6.84 (b) no

3 Range and domain by completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{12}{x^2 - 4x + 6}$ for all real x.
Find the greatest value of f(x), the value of x at which it occurs, and the range of f.

$x^2 - 4x + 6 = (x - 2)^2 + 2$.
$(x - 2)^2 \geq 0$, so the denominator is at least 2, with equality at x = 2.
The denominator is always positive, so f(x) > 0.

Greatest f = $\frac{12}{2} = 6$ at x = 2.

Answer: greatest value 6 at x = 2; range $0 < f(x) \leq 6$

4 Velocity-time graph with a return (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the velocity-time graph of a particle P moving on a straight line (velocity in m s^{-1}, time in s). At time 15 s, P is back at its starting point.
Find (a) the value of W, (b) the greatest distance of P from its starting point.

Net area = 0 (back at start): ½ × 3 × 10 + ½(10 − W)(6) − W(6) = 0
15 + 30 − 3W − 6W = 0, so 9W = 45
W = 5

v = 0 where the line from (3, 10) to (9, −5) crosses the axis: $t = 3 + 6 \times \frac{10}{15} = 7$

Greatest distance = area of triangle = ½ × 7 × 10
Answer: (a) W = 5 (b) 35 m

5 Derivative of the inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve is given by $x = y^3 + 2y^2 + 2y$.
Find dy/dx and d^2y/dx^2 when y = −1.

$dx/dy = 3y^2 + 4y + 2 = 1$ when y = −1, so $dy/dx = \frac{1}{dx/dy} = 1$

$d^2x/dy^2 = -2$ when y = −1

$d^2y/dx^2 = -\frac{d^2x}{dy^2/(dx/dy)^3} = -\frac{-2}{1^3}$

Answer: dy/dx = 1, $d^2y/dx^2 = 2$

6 Arithmetic series from a sum formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence has $u_5 = 31$ and $S_8 = 220$.

(a) Find the first term and common difference.

(b) Find the least n for which $S_n > 264$.

u_5: $a + 4d = 31$. S_8: $8(2a + 7d)/2 = 220$, so $8a + 28d = 220$.

Solving: $a = 3$, $d = 7$.

$S_n = \frac{n}{2}(6 + 7n - 7) = \frac{n(7n - 1)}{2}$.

Test: $S_8 = 220 \leq 264$, $S_9 = 279 > 264$.

Answer: (a) a = 3, d = 7 (b) n = 9

7 PMCC from summary data and a hypothesis test (AQA, Edexcel, OCR, MEI, Eduqas)

For 20 pairs of values, $S_{xx} = 784$, $S_{yy} = 841$ and $S_{xy} = 256$.

(a) Calculate the product moment correlation coefficient r.

(b) The critical value for a two-tailed test at the 1% level with n = 20 is 0.5614. Carry out the test of H_0: $\rho = 0$ against H_1: $\rho \neq 0$.

$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}} = \frac{256}{\sqrt{(784 \times 841)}} = \frac{256}{812} = 0.3153$

H_0: $\rho = 0$, H_1: $\rho \neq 0$

$|r| = 0.3153 <$ the critical value 0.5614

Do not reject H_0: there is insufficient evidence of correlation between the variables

Answer: (a) r = 0.315 (b) do not reject H_0

8 Ladder against a smooth wall: how high can a person climb (AQA, Edexcel, OCR, MEI)

A uniform ladder AB of length 5 m and weight 300 N has A on rough ground ($\mu = 0.35$) and B against a smooth vertical wall. It makes angle θ with the ground, $\tan\theta = 2$.

A man of weight 400 N climbs the ladder. How far along the ladder can he go before it slips?

Wall reaction S horizontal, ground reaction R vertical, friction F = S. Moments about A: $S \times L \sin\theta = W \times \frac{L}{2}\cos\theta$

So $S = \frac{W}{2\tan\theta}$. Unloaded, $R = W$: need $\frac{W}{2\tan\theta} \leq \mu W$, so $\mu \geq \frac{1}{2 \times 2} = 0.25$

With the man at distance x: $R = 700$, and moments about A give $S L \sin\theta = (300 \times 2.5 + 400x)\cos\theta$

Limiting: $S = \mu R = 0.35 \times 700 = 245$, so $245 \times 5 \times 2 = 300 \times 2.5 + 400x$

$400x = 2450 - 750$, so $x = 4.25$

Answer: 4.25 m

9 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$\frac{dy}{dx} = \frac{9x^2}{y}$, $y > 0$, and $y = 4$ when $x = 0$.

Find y in terms of x, and the value of y when x = 4.

$\int y\,dy = \int 9x^2\,dx$ gives $\frac{y^2}{2} = 3x^3 + c$

$x = 0$, $y = 4$: $c = 8$, so $y^2 = 6x^3 + 16$

$x = 4$: $y^2 = 400$

Answer: $y = \sqrt{6x^3 + 16}$; y = 20

10 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x + 4| > |3x - 4|$.

Square both sides: $(2x + 4)^2 > (3x - 4)^2$.

$(-x + 8)(5x) > 0$.

Critical values $x = 8$ and $x = 0$.

The quadratic has negative coefficient of x^2, so choose the inner region.

Answer: 0 < x < 8