Day 75 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Double angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cos 2\theta + \sin\theta = 0$
for $0 \leq \theta < 360°$.

Use $\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta - \sin\theta - 1 = 0$
$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{2}$ or $\sin\theta = 1$

Using the CAST diagram for $0 \leq \theta < 360°$
Answer: θ = 90°, 210°, 330°

2 Binomial: finding p from a ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(8, p)$. It is given that $P(X = 2) = \frac{7}{6} \times P(X = 1)$.

(a) Find the value of p.
(b) Find $P(X = 0)$, correct to 3 significant figures.

$\frac{56}{2}\, p^2(1-p)^6 = \frac{7}{6} \times 8\, p(1-p)^7$

Divide by $p(1-p)^6$: $3.5\, p = \frac{7}{6}(1-p)$

$(3.5 + \frac{7}{6})p = \frac{7}{6}$

$p = \frac{1}{4}$

$P(X = 0) = (1-p)^8 = (\frac{3}{4})^8 = 0.100$

Answer: (a) $p = \frac{1}{4}$ (b) 0.100

3 Perpendicular lines and area (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the diagram, line l_1 passes through A and B.
Line l_2 is perpendicular to l_1 at B and meets the
x-axis at C. Find the coordinates of C and the area of triangle ABC.

Gradient of $l_1 = \frac{9-7}{3+3} = \frac{1}{3}$.
Gradient of $l_2 = -3$. Equation: $y - 9 = -3(x - 3)$.
$y = 0$: $x = 6$, so C(6, 0).
$AB = 2\sqrt{10}$, $BC = 3\sqrt{10}$: area = ½ × AB × BC.
Answer: C(6, 0); area 30

4 Horizontal force on a rough slope: range of values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of weight 55 N rests on a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{5}{12}$ and $\mu = 0.2$.

A horizontal force H N, in the vertical plane through a line of greatest slope, holds it in equilibrium. Find the range of possible values of H.

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$. Normal reaction: $R = 55\cos\alpha + H\sin\alpha$

About to slide down: $H\cos\alpha + \mu R = 55\sin\alpha$, so $H(\frac{12}{13} + 0.2 \times \frac{5}{13}) = 55(\frac{5}{13} - 0.2 \times \frac{12}{13})$

$H_{min} = 11$

About to slide up: $H\cos\alpha = 55\sin\alpha + \mu R$, so $H(\frac{12}{13} - 0.2 \times \frac{5}{13}) = 55(\frac{5}{13} + 0.2 \times \frac{12}{13})$

$H_{max} = 37$
Answer: $11 \leq H \leq 37$

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5 Range and domain by completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{12}{2x^2 - 12x + 24}$ for all real x.

Find the greatest value of f(x), the value of x at which it occurs, and the range of f.

$2x^2 - 12x + 24 = 2(x - 3)^2 + 6$.

$(x - 3)^2 \geq 0$, so the denominator is at least 6, with equality at x = 3.

The denominator is always positive, so f(x) > 0.

Greatest f = $\frac{12}{6} = 2$ at x = 3.

Answer: greatest value 2 at x = 3; range $0 < f(x) \leq 2$

6 Area between a curve and a line (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curves y = 6 cos x and y = 6 cos 2x meet at x = 0 and

at x = $\frac{2\pi}{3}$. Find the exact area enclosed between them

for $0 \leq x \leq \frac{2\pi}{3}$.

For $0 < x < \frac{2\pi}{3}$, cos x > cos 2x

Area = $6\int_0^{2\pi/3} (\cos x - \cos 2x)\, dx = 6[\sin x - \frac{\sin 2x}{2}]$

$= 6[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}]$

Answer: $\frac{9}{2}\sqrt{3}$

7 Replacing an observation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

20 recorded times (seconds) have mean 22 and standard deviation 8.

One value was recorded as 42 but should have been 62.

Find the correct mean and the correct standard deviation.

$\Sigma x = 20 \times 22 = 440$, so corrected $\Sigma x = 440 - 42 + 62 = 460$

Correct mean = $\frac{460}{20} = 23.00$

$\Sigma x^2 = 20(64 + 484) = 10960$, corrected $\Sigma x^2 = 10960 - 1764 + 3844 = 13040$

Variance = $\frac{13040}{20} - 23.000^2 = 123.00$, $\sigma = 11.09$

Answer: mean 23.0, standard deviation 11.1

8 Arithmetic series modelling (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first row of a theatre has 17 seats. Each row has 6 more seats than the row in front. Find

(a) the number of seats in row 35,

(b) the least number of rows for at least 1696 seats.

Arithmetic: a = 17, d = 6.

(a) $u_{35} = 17 + 34 \times 6 = 221$.

(b) $S_n = \frac{n}{2}(34 + 6n - 6) = \frac{n(6n + 28)}{2} \geq 1696$.

$S_{21} = 1617 < 1696$, $S_{22} = 1760 \geq 1696$.

Answer: (a) 221 (b) 22 rows

9 Closest approach of moving points (Edexcel, OCR, MEI, Eduqas, CIE (vectors in kinematics))

Two drones P and Q move so that, t seconds after launch, their position vectors in metres are

$\mathbf{r}_P = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} + t\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix} -7 \\ -1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$.

Find the time at which they are closest and the exact minimum distance between them.

$\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix} -8 \\ -5 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$

Distance$^2 = |\mathbf{r}_Q - \mathbf{r}_P|^2$, minimised when $(\mathbf{r}_Q - \mathbf{r}_P) \cdot \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = 0$

$-10 + t(5) = 0$, so $t = 2$

Then $\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix} -8 \\ -1 \\ 2 \end{pmatrix}$, with length $\sqrt{69}$

Answer: t = 2, minimum distance = $\sqrt{69}$ m

10 Sinusoidal velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a straight line. At time t seconds its velocity is v m s^{-1}, where $v = 5\sin(\frac{\pi t}{3})$ for $t \geq 0$.

Find (a) the greatest magnitude of the acceleration of P, (b) the distance P travels between the first two times at which it is at rest. Give (b) to 3 s.f.

$a = \frac{dv}{dt} = \frac{5}{3}\pi\cos(\frac{\pi t}{3})$, so the greatest magnitude is $\frac{5}{3}\pi$

$v = 0$ when $\sin(\frac{\pi t}{3}) = 0$: $t = 0$ and $t = 3$

$s = \int$ from 0 to 3 of $5\sin(\frac{\pi t}{3})\,dt = [-\frac{5 \times 3}{\pi}\cos(\frac{\pi t}{3})]$ from 0 to 3

$= \frac{15}{\pi}(1 - (-1)) = \frac{30}{\pi}$

Answer: (a) $\frac{5}{3}\pi$ m s^{-2} (b) 9.55 m