Day 71 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Stone thrown from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 15 m s^{-1} from the top of a cliff. The top of the cliff is 18.4 m above the sea.
Find (a) the time taken for the stone to reach the sea, (b) the total distance travelled by the stone, to 3 s.f. Take g = 9.8 m s^{-2}.

Take upwards as positive: $-18.4 = 15t - 4.9t^2$, so $4.9t^2 - 15t - 18.4 = 0$

$t = \frac{15 + \sqrt{(15^2 + 19.6 \times 18.4)}}{9.8} = \frac{15 + 24.2}{9.8} = 4$

Height risen above the cliff top = $\frac{15^2}{(2 \times 9.8)} = 11.5$ m

Total distance = 2 × 11.5 + 18.4

Answer: (a) 4 s (b) 41.4 m

2 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 2x + 4 and $g(x) = x^2 - 2$.
Solve fg(x) = gf(x).

$fg(x) = 2(x^2 - 2) + 4 = 2x^2$.

$gf(x) = (2x + 4)^2 - 2 = 4x^2 + 16x + 14$.

Equate: $2x^2 + 16x + 14 = 0$.

Solve the quadratic: x = −7 or x = −1.

Answer: x = −7 or x = −1

3 Logarithmic equations with ln (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve 2 ln x = ln(x + 4) + ln 2,
stating clearly why one root is rejected.

$\ln x^2 = \ln(2(x + 4))$, so $x^2 = 2x + 8$

$x^2 - 2x - 8 = 0$, $(x - 4)(x + 2) = 0$

x = −2 is rejected since ln x is undefined for x < 0

Answer: x = 4

4 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
x = 2 + 3cos t, y = 1 + 3sin t.
Find its Cartesian equation.

$\cos t = \frac{x - 2}{3}$, $\sin t = \frac{y - 1}{3}$.

Use $\cos^2 t + \sin^2 t = 1$:

$(x - 2)^2 + (y - 1)^2 = 9$.

Answer: $(x - 2)^2 + (y - 1)^2 = 9$

5 Velocity-time graph with a return (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the velocity-time graph of a particle P moving on a straight line (velocity in m s^{-1}, time in s). At time 13 s, P is back at its starting point.
Find (a) the value of W, (b) the greatest distance of P from its starting point.

Net area = 0 (back at start): ½ × 2 × 6 + ½(6 − W)(6) − W(5) = 0

6 + 18 − 3W − 5W = 0, so 8W = 24

W = 3

v = 0 where the line from (2, 6) to (8, −3) crosses the axis: $t = 2 + 6 \times \frac{6}{9} = 6$

Greatest distance = area of triangle = ½ × 6 × 6

Answer: (a) W = 3 (b) 18 m

6 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2 - t$, $y = 2t^3 + 4t$.
Find the equation of the tangent at the point where $t = 1$,
in the form $y = mx + c$.

$dx/dt = 6t - 1$, $dy/dt = 6t^2 + 4$

At $t = 1$: $dy/dx = \frac{10}{5} = 2$; point $(2, 6)$

$y - 6 = 2(x - 2)$

Answer: $y = 2x + 2$

7 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The size of a rod is $N(110, 4^2)$ mm. A rod is 'large' if it exceeds 114.
(a) Find the probability that a rod is large.
(b) 13 rods are chosen at random. Find the probability that at least 3 are large.

(a) $P(X > 114) = P(Z > 1.000) = 0.1587$
(b) Let Y = number large, $Y \sim B(13, 0.1587)$
$P(Y \geq 3) = 1 - P(Y \leq 2) = 0.341$
Answer: (a) 0.159 (b) 0.341

8 Tangent and axes triangle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tangent to $y = x\sqrt{2x + 8}$ at the point where $x = 4$
meets the axes at A and B. Find the exact area of triangle OAB,
where O is the origin.

At $x = 4$: $y = 16$. Differentiate: gradient $= 5$
Tangent: $y - 16 = 5(x - 4)$, so $y = 5x - 4$

y-intercept -4; x-intercept $\frac{4}{5}$

Area $= \frac{1}{2} \times |-4| \times |\frac{4}{5}|$

Answer: $\frac{8}{5}$

9 Regression line from summations (AQA, Edexcel, OCR, MEI)

For $n = 12$ pairs of values: $\Sigma x = 180$, $\Sigma y = 540$, $\Sigma x^2 = 2884$, $\Sigma xy = 8560$.
(a) Find the equation of the regression line of y on x.
(b) The x values lie between 11 and 20. Estimate y when $x = 19$ and comment on its reliability.

$S_{xx} = 2884 - \frac{180^2}{12} = 184$

$S_{xy} = 8560 - \frac{180 \times 540}{12} = 460$

$b = \frac{S_{xy}}{S_{xx}} = \frac{460}{184} = \frac{5}{2}$

$a = \bar{y} - b\,\bar{x} = 45 - (\frac{5}{2})(15) = 7.5$

$y = 7.5 + 2.5x$
At $x = 19$: $y = 55$. This is extrapolation (outside the data range), so may be unreliable
Answer: (a) $y = 7.5 + 2.5x$ (b) 55, unreliable (extrapolation)

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10 Transformations of y = x^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe a sequence of two transformations that maps the curve $y = x^2$ onto the curve $y = 5x^2 + 30x + 48$.

Complete the square: $y = 5(x + 3)^2 + 3$.

Step 1: $y = x^2 \rightarrow y = 5x^2$: a stretch, scale factor 5, parallel to the y-axis.

Step 2: replace x by (x + 3) and add 3: translation by $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$.

Answer: a stretch, scale factor 5, parallel to the y-axis, then translation $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$