**Day 70** **A Level Maths: S Tier 10-a-day: Model Solutions**

## 1 Tree diagram with an unknown probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows two games played in turn.

Given the second is a W, P(first was a W) = $\frac{1}{7}$. Find x.

P(W then W) = 0.2x, P(second W) = 0.2x + 0.6(1 − x)

P(first W | second W) = $\frac{0.2x}{0.2x + 0.6(1 - x)} = \frac{1}{7}$

$0.2x = \frac{1}{7}(0.2x + 0.6 - 0.6x)$

Solve the linear equation for x: $x = \frac{1}{3}$

**Answer: $x = \frac{1}{3}$**

## 2 Volumes of revolution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region bounded by $y = xe^{-x}$, the x-axis and the lines
$x = 0$ and $x = 2$ is rotated through $2\pi$ about the x-axis.
Find the exact volume of the solid formed.

$V = \pi\int_0^2 y^2\,dx = \pi\int_0^2 (xe^{-x})^2\,dx$

Simplify the integrand, then integrate

$V = \pi \times (\frac{1}{4} - \frac{13}{4}e^{-4})$

**Answer: $\frac{1}{4}\pi - \frac{13}{4}\pi e^{-4}$**

## 3 Proof by deduction (divisibility) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $(n + 4)^3 - n^3 - 64$ is a multiple of 12 for every integer n.

$(n + 4)^3 = n^3 + 12n^2 + 48n + 64$.
So the expression $= 12n^2 + 48n$
$= 12n(n + 4)$, a multiple of 12.
**Answer: Proved**

## 4 Distance travelled with a change of direction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time t seconds its velocity is v m $s^{-1}$, where
$v = t^2 - 8t + 12$ for $0 \le t \le 7$
Find (a) the times at which P is at instantaneous rest, (b) the total distance travelled by P in the first 7 seconds.

$v = (t - 2)(t - 6)$, so $v = 0$ when $t = 2$ and $t = 6$
$v > 0$ for $0 < t < 2$, $v < 0$ for $2 < t < 6$, $v > 0$ for $t > 6$: direction changes at each

$s = \int v\,dt = \frac{1}{3}t^3 - 4t^2 + 12t$

Distances: 0 to 2: $\frac{32}{3}$; 2 to 6: $\frac{32}{3}$; 6 to 7: $\frac{7}{3}$

Total = $\frac{32}{3} + \frac{32}{3} + \frac{7}{3}$

**Answer: (a) $t = 2$, $t = 6$ (b) $\frac{71}{3}$ m**

## 5 Proof by exhaustion (modular) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $8k = n^2 + 1$ has no integer solutions (k and n integers).

Consider $n = 8q + r$, with $r = 0, 1, ..., 7$.
Squaring, $n^2 \equiv r^2 \pmod 8$, so:
$0 \to 0$; $1 \to 1$; $2 \to 4$; $3 \to 1$; $4 \to 0$; $5 \to 1$; $6 \to 4$; $7 \to 1$ ($r \to n^2 \bmod 8$)
So $n^2 + 1 \equiv 1$ or 2 or 5 (mod 8).
No residue is 0, so $n^2 + 1$ is never a multiple of 8.
**Answer: Proved**

## 6 Stationary points of trigonometric curves (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the stationary points of
$y = 3x - 6\sin x$ for $0 \le x \le 2\pi$,
and determine their nature.

$dy/dx = 3 - 6\cos x = 0$, so $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$

$d^2y/dx^2 = 6\sin x$
Its sign at each point decides maximum or minimum

**Answer: $(\frac{\pi}{3}, \pi - 3\sqrt{3})$ minimum; $(\frac{5\pi}{3}, 5\pi + 3\sqrt{3})$ maximum**

## 7 Triangular pdf from a diagram (CIE)

The diagram shows the pdf of X.
Find (a) k, (b) E(X), (c) the median of X (3 s.f.).

(a) Area of triangle = ½ × 10 × k = 1, so $k = \frac{1}{5}$

(b) The mean of a triangular distribution is $\frac{0 + 9 + 10}{3} = \frac{19}{3}$

$F(9) = \frac{9}{10} = \frac{9}{10} \ge \frac{1}{2}$, so the median is in the left part

$F(x) = \frac{x^2}{90} = \frac{1}{2}$, so $x = \sqrt{45} = 6.708$

**Answer: (a) $k = \frac{1}{5}$ (b) $E(X) = \frac{19}{3}$ (c) 6.71**

## 8 Hinged rod supported by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (3 m, 80 N) is hinged at A to a vertical wall and held horizontal by a string BC, C on the wall above A, with $\tan\alpha = 2.4$, where $\alpha$ is the angle between string and rod. A 40 N particle hangs at P, AP = 1.5 m.
Find (a) the tension, (b) the magnitude and direction of the hinge force (3 s.f. and 1 d.p.).

$\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$. Moments about A: $T\sin\alpha \times 3 = 80 \times 1.5 + 40 \times 1.5$

$T \times \frac{12}{13} \times 3 = 180$, so $T = 65$

Horizontal: $H = T\cos\alpha = 25$. Vertical: $V + T\sin\alpha = 80 + 40$, so $V = 60$

Force $= \sqrt{(25^2 + 60^2)} = 65.0$, angle $= \tan^{-1}(\frac{60}{25}) = 67.4°$ above the horizontal

**Answer: (a) 65 N (b) 65.0 N at 67.4° above the horizontal (away from the wall)**

## 9 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(−6, 6) and B(2, 10). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c - (-6))^2 + 36 = (c - 2)^2 + 100$.
Expand and solve: $16c = 32$, so $c = 2$.
Area $= \frac{1}{2}|(x_A - c)y_B - (x_B - c)y_A|$
$= \frac{1}{2}|(-8) \times 10 - 0 \times 6| = 40$.
**Answer: C(2, 0); area 40**

## 10 Area under a parametric curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 4 \sin t$, $y = 6 \cos t$, for $0 \leq t \leq \frac{\pi}{2}$. Find the exact area bounded by the curve and the coordinate axes.

$dx/dt = 4 \cos t$

Area $= \int_0^{\pi/2} 6 \cos t \times 4 \cos t \, dt = 24 \int_0^{\pi/2} \cos^2 t \, dt$

$\cos^2 t = \frac{1}{2}(1 + \cos 2t)$: integral $= 24 \times \frac{\pi}{4}$

**Answer: $6\pi$**