Day 7 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Integrals of the form f'(x)/f(x) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$$\int_0^{\pi/2} \frac{\cos x}{2 + \sin x}\,dx$$

Numerator is the derivative of the denominator
Integral = [ln(2 + sin x)]
= ln 3 − ln 2
Answer: −ln 2 + ln 3

2 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 32 and upper quartile 50. An outlier is a value more than 1.5 × IQR beyond the nearest quartile.
(a) Find the range of values that are not outliers.
(b) 8 observations have mean 38. The largest is 89, an outlier, and it is removed. Find the new mean.

IQR = 50 − 32 = 18, so 1.5 × IQR = 27
Lower limit = 32 − 27 = 5, upper limit = 50 + 27 = 77
Σx = 8 × 38 = 304, so without 89: Σx = 215
New mean = $\frac{215}{7}$
Answer: (a) 5 to 77 (b) 30.71

3 Arithmetic series modelling (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A pile of logs has 40 logs in its bottom row. Each row above has 2 fewer logs than the row below, and the pile has as many rows as possible, each with at least one log. Find the number of rows and the total number of logs.

Row n has 40 − 2(n − 1) = −2n + 42 logs; need this ≥ 1.
2(n − 1) ≤ 39, so n ≤ $\frac{41}{2}$: 20 rows.
Top row = 2 logs. S = $\frac{20}{2}$(40 + 2) = 420.
Answer: 20 rows, 420 logs

4 Velocity-time graph with a return (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the velocity-time graph of a particle P moving on a straight line (velocity in m s^{-1}, time in s). At time 17 s, P is back at its starting point.
Find (a) the value of W, (b) the greatest distance of P from its starting point.

Net area = 0 (back at start): ½ × 4 × 10 + ½(10 − W)(6) − W(7) = 0
20 + 30 − 3W − 7W = 0, so 10W = 50
W = 5
v = 0 where the line from (4, 10) to (10, −5) crosses the axis: t = 4 + 6 × $\frac{10}{15}$ = 8
Greatest distance = area of triangle = ½ × 8 × 10
Answer: (a) W = 5 (b) 40 m

5 Sectors: perimeter and area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius r cm and angle θ radians, where θ < 2π.
Its perimeter is 46 cm and its area is 132 cm^2.
Find the two possible pairs of values of r and θ.

Arc = rθ, so 2r + rθ = 46 and $\frac{1}{2}r^2\theta$ = 132
rθ = 46 − 2r, so $\frac{1}{2}$r(46 − 2r) = 132
r^2 − 23r + 132 = 0, so (r − 11)(r − 12) = 0
r = 11: rθ = 24, θ = $\frac{24}{11}$ r = 12: rθ = 22, θ = $\frac{11}{6}$
Answer: r = 11, θ = $\frac{24}{11}$ or r = 12, θ = $\frac{11}{6}$

6 Indices: quadratic in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2^{2x+1} - 17 \times 2^x + 8 = 0$.

Let $y = 2^x$. Then $2y^2 - 17y + 8 = 0$.

$2(y - \frac{1}{2})(y - 8) = 0$, so $y = \frac{1}{2}$ or $y = 8$.

$2^x = \frac{1}{2}$ gives $x = -1$; $2^x = 8$ gives $x = 3$.

Answer: $x = -1$ or $x = 3$

7 Non-uniform rod: centre of mass and tilting (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB of length 5 m and mass 8 kg rests on supports at C and D, where AC = 2 m and AD = 4 m, with $R_C = \frac{1}{3} R_D$.

Find (a) the distance of the centre of mass from A. A particle of mass m kg at B puts the rod on the point of tilting about D. (b) Find m.

$R_C = \frac{1}{3} R_D$. Moments about A: $R_C \times 2 + R_D \times 4 = 8g \times AG$

$R_D(\frac{1}{3} \times 2 + 4) = 8g \times AG$ with $R_D(1 + \frac{1}{3}) = 8g$, so $AG = \dfrac{\frac{1}{3} \times 2 + 4}{\frac{4}{3}} = 3.5$

Tilting about D: $R_C = 0$. Moments about D: $mg(5 - 4) = 8g(4 - 3.5)$

$m = \dfrac{8 \times 0.5}{1} = 4$

Answer: (a) 3.5 m (b) 4

8 Points of inflection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the point of inflection of

$y = (x - 1)e^{-2x}$,

justifying that it is a point of inflection.

$dy/dx = e^{-2x}(1 - 2(x - 1))$

$d^2y/dx^2 = 2e^{-2x}(2(x - 1) - 2) = 0$, so $x = 2$

d^2y/dx^2 changes sign either side of $x = 2$

$y = e^{-4}$

Answer: $(2, e^{-4})$

9 Pdf of the form a + bx with a given mean (CIE)

The pdf of X is $f(x) = a + bx$ for $0 \le x \le 4$, and 0 otherwise, where a and b are constants. $E(X) = \frac{12}{5}$.

(a) Find a and b.
(b) Find $P(X < 2)$.

Total area = 1: $4a + 8b = 1$

$E(X) = \int x(a + bx)\,dx = 8a + \frac{64}{3} b = \frac{12}{5}$

Solving the simultaneous equations: $a = \frac{1}{10}$, $b = \frac{3}{40}$

$P(X < 2) = \int f(x)\,dx$ from 0 to 2 $= \frac{1}{10} \times 2 + \frac{3}{40} \times 2$

$= \frac{7}{20}$

Answer: (a) $a = \frac{1}{10}$, $b = \frac{3}{40}$ (b) $\frac{7}{20}$

10 Circle through three points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through P(6, 5), Q(2, −3) and R(−3, 2). Find its equation.

Centre is on the perpendicular bisector of each chord.
Bisector of PQ: $-4x - 8y = -24$.
Bisector of QR: $-5x + 5y = 0$.
Solving: centre (2, 2).
$r^2 = (6 - 2)^2 + (5 - 2)^2 = 25$.
Answer: $(x - 2)^2 + (y - 2)^2 = 25$