Day 68 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Stratified sample then probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A school has 80 students in Year 10, 160 in Year 11 and 60 in the sixth form.
A stratified sample of 30 students is taken.
(a) Find the number sampled from each group.
(b) Two students are chosen at random from the sample. Find the probability they are from the same group.

Sample sizes: $30 \times \frac{80}{300}$, $30 \times \frac{160}{300}$, $30 \times \frac{60}{300} = 8, 16, 6$

P(same) = $8 \times 7 + 16 \times 15 + 6 \times 5/30 \times 29$

$= \frac{326}{870} = 0.375$

Answer: (a) 8, 16, 6 (b) 0.375

2 Double angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3\cos 2\theta + 9\sin\theta + 3 = 0$
for $0 \leq \theta < 2\pi$, giving answers in terms of π.

Use $\cos 2\theta \equiv 1 - 2\sin^2\theta$: $6\sin^2\theta - 9\sin\theta - 6 = 0$

$3(2\sin\theta + 1)(\sin\theta - 2) = 0$

$\sin\theta = -\frac{1}{2}$ or $\sin\theta = 2$ (impossible, $|\sin\theta| \leq 1$)

Principal values and symmetry in $[0, 2\pi)$

Answer: $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

3 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation $y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes $x = 3$ and $y = 2$, so $p = -3$ and $q = 2$.

$y = \frac{a}{x - 3} + 2$.

P(5, 4): $4 = \frac{a}{2} + 2$, so $a = 4$.

Answer: $a = 4$, $p = -3$, $q = 2$

4 Particle in equilibrium on two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P of weight 45 N hangs in equilibrium from two light strings AP and BP attached to a horizontal ceiling at A and B (see diagram), where $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{4}{3}$.

Find (a) the tension in each string, (b) the greatest weight P could have if neither tension may exceed 76 N.

Horizontally: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_1 \times \frac{4}{5} = T_2 \times \frac{3}{5}$

Vertically: $T_1 \sin\alpha + T_2 \sin\beta = 45$, and $T_2 = \frac{4}{3} T_1$

$T_1(\frac{3}{5} + \frac{4}{3} \times \frac{4}{5}) = 45$, so $T_1 = 27$ and $T_2 = 36$

The tension in BP is the larger: it is 36 when W = 45

Tensions are proportional to W, so $W_{max} = 45 \times \frac{76}{36} = 95$

Answer: (a) AP: 27 N, BP: 36 N (b) 95 N

5 Remainder theorem (quadratic divisor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when x^8 is divided by
$(x - 1)(x - 3)$.

$x^8 = (x - 1)(x - 3)Q(x) + px + q$.
$x = 1$: $1 = p + q$.
$x = 3$: $6561 = 3p + q$.
Subtract: $-2p = -6560$, so $p = 3280$ and $q = -3279$.
Answer: $3280x - 3279$

6 Relative motion of two projectiles (AQA, Edexcel, OCR, MEI, Eduqas)

Particles A and B are projected from a point O at the same instant, with initial velocities $(18\mathbf{i} + 31\mathbf{j})$ m s^{-1} and $(14\mathbf{i} + 34\mathbf{j})$ m s^{-1} respectively (i horizontal, j vertically upwards).
Both are still in flight after 4 s. Find (a) the distance AB at this time, (b) the angle AB makes with the horizontal, to 1 d.p.

Position of A: $18t\mathbf{i} + (31t - 4.9t^2)\mathbf{j}$; position of B: $14t\mathbf{i} + (34t - 4.9t^2)\mathbf{j}$
Subtracting, the $4.9t^2$ terms cancel: AB = $t(4\mathbf{i} - 3\mathbf{j})$
After 4 s: AB = $(16\mathbf{i} - 12\mathbf{j})$
Distance = $\sqrt{(16^2 + 12^2)} = 20$
The direction of AB is that of $(4\mathbf{i} - 3\mathbf{j})$, which does not depend on t: tan angle = $\frac{3}{4}$
Answer: (a) 20 m (b) 36.9°

7 Tangent and axes triangle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tangent to $y = \frac{2}{x^2 + 3}$ at the point where $x = 3$
meets the axes at A and B. Find the exact area of triangle OAB,
where O is the origin.

At $x = 3$: $y = \frac{1}{6}$. Differentiate: gradient = $-\frac{1}{12}$
Tangent: $y - \frac{1}{6} = (-\frac{1}{12})(x - 3)$, so $y = -\frac{1}{12}x + \frac{5}{12}$
y-intercept $\frac{5}{12}$; x-intercept 5
Area = $\frac{1}{2} \times |\frac{5}{12}| \times |5|$
Answer: $\frac{25}{24}$

8 Tree diagram with an unknown probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows two games played in turn.
Given the second is a W, P(first was a W) = $\frac{3}{11}$. Find x.

P(W then W) = $0.6x$, P(second W) = $0.6x + 0.4(1 - x)$
P(first W | second W) = $\frac{0.6x}{0.6x + 0.4(1 - x)} = \frac{3}{11}$
$0.6x = \frac{3}{11}(0.6x + 0.4 - 0.4x)$
Solve the linear equation for x: $x = \frac{1}{5}$
Answer: $x = \frac{1}{5}$

9 Perpendicular from a point to a line (3D) (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix} + t\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$.

The point P has coordinates $(-6, -7, 0)$.

Find the coordinates of the foot F of the perpendicular from P to l, and the exact distance PF.

F = (a + td) for some t, so $\mathbf{FP} = \begin{pmatrix} -8 \\ -4 \\ 2 \end{pmatrix} - t\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$\mathbf{FP} \cdot \mathbf{d} = 0$: $4 - t(2) = 0$, so $t = 2$

F = $(0, -1, -2)$

$\mathbf{PF} = \begin{pmatrix} -6 \\ -6 \\ 2 \end{pmatrix}$, so $PF = \sqrt{76}$

Answer: F = (0, −1, −2), PF = $2\sqrt{19}$

10 Arithmetic series from a sum formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The sum of the first n terms of a sequence is

$S_n = 4n^2 + 7n$.

(a) Find an expression for u_n.

(b) Find the least n for which $u_n > 311$.

$u_n = S_n - S_{n-1} = 4n^2 + 7n - [4(n - 1)^2 + 7(n - 1)]$

$= 8n + 3$.

$u_1 = S_1 = 11$ also fits, so $u_n = 8n + 3$ for all n.

$8n + 3 > 311 \Rightarrow n > \frac{77}{2}$, so n = 39.

Answer: (a) $u_n = 8n + 3$ (b) n = 39