Day 67 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{6x + 1} = x - 1$.

Square both sides: $6x + 1 = (x - 1)^2$.

$x^2 - 8x = 0$, so $x = 0$ or $x = 8$.

Check in $\sqrt{6x + 1} = x - 1$: $x = 8$ gives $7 \geq 0$ (valid).

$x = 0$ gives right-hand side $-1 < 0$, so it is rejected.

Answer: $x = 8$

2 Sectors: perimeter and area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius r cm and angle θ radians, where $\theta < 2\pi$.

Its perimeter is 40 cm and its area is 91 cm^2.

Find the two possible pairs of values of r and θ.

Arc = $r\theta$, so $2r + r\theta = 40$ and $\frac{1}{2}r^2\theta = 91$

$r\theta = 40 - 2r$, so $\frac{1}{2}r(40 - 2r) = 91$

$r^2 - 20r + 91 = 0$, so $(r - 7)(r - 13) = 0$

$r = 7$: $r\theta = 26$, $\theta = \frac{26}{7}$ $r = 13$: $r\theta = 14$, $\theta = \frac{14}{13}$

Answer: $r = 7$, $\theta = \frac{26}{7}$ or $r = 13$, $\theta = \frac{14}{13}$

3 Cubic graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the cubic $y = f(x)$, which touches the x-axis once. Find f(x), giving your answer in expanded form.

Crosses at $x = -2$ and touches at $x = -3$, so $f(x) = k(x + 2)(x + 3)^2$.

At (0, 18): $18 = k(2)(3)^2 = 18k$, so $k = 1$.

$f(x) = (x + 2)(x + 3)^2$, then expand.

Answer: $f(x) = x^3 + 8x^2 + 21x + 18$

4 Three-stage journey (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train accelerates uniformly from rest at 1.4 m s^{-2} to a speed of V m s^{-1}, travels at this speed, then decelerates uniformly at 1.75 m s^{-2} to rest.

The journey takes 54 s and covers 630 m. Find V and the time spent at constant speed.

Time accelerating = $\frac{V}{1.4}$, time decelerating = $\frac{V}{1.75}$, so time at constant speed = $54 - \frac{V}{1.4} - \frac{V}{1.75}$

Area under the speed-time graph: $630 = 54V - \frac{V^2}{2.8} - \frac{V^2}{3.5}$

Multiply by 2ab: $3.15V^2 - 264.6V + 3087 = 0$

$V = 14$ or $V = 70$

$V = 70$ gives time at constant speed -36 s, which is negative, so $V = 14$

Time at constant speed = $54 - 10 - 8 = 36$ s

Answer: $V = 14$, 36 s

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5 Waiting for the r-th success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each shot Amira takes scores with probability $\frac{2}{5}$, independently.

(a) Find the probability that her third score comes on shot number 8.
(b) Find the probability that her third score comes on or before shot number 8.

(a) third score on shot 8: exactly 2 scores in the first 7 shots, then a score

$C(7, 2)(\frac{2}{5})^2(\frac{3}{5})^5 \times \frac{2}{5}$

$= 0.105$

(b) On or before shot 8 means at least 3 scores in 8 shots: $X \sim B(8, \frac{2}{5})$

$P(X \geq 3) = 1 - P(X \leq 2) = 0.685$

Answer: (a) 0.105 (b) 0.685

6 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 807
inclusive that are divisible by 6 or by 10, but not by both.

Multiples of 6: 134 terms (6 to 804): sum = 54270.
Multiples of 10: 80 terms: sum = 32400.
Multiples of both, i.e. of 30: 26 terms: sum = 10530.
Required sum = 54270 + 32400 − 2 × 10530.

Answer: 65610

7 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_1^e \frac{\ln x}{x^3}\,dx$

Let $u = \ln x$ and $dv/dx = x^{-3}$, so $du/dx = \frac{1}{x}$, $v = -\frac{1}{2x^2}$

Integral $= [-\frac{\ln x}{2x^2}] + \int \frac{1}{2x^3}\,dx$

Answer: $\mathbf{\frac{1}{4} - \frac{3}{4}e^{-2}}$

8 Speed decaying exponentially (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a straight line. The velocity of P is v m s^{-1} and its acceleration is $-0.25v$ m s^{-2}. At t = 0, v = 10.
Find (a) the time taken for the speed of P to halve, to 3 s.f., (b) the distance P travels in this time.

$\frac{dv}{dt} = -0.25v$: $\int \frac{dv}{v} = -0.25\int dt$, so $\ln v = -0.25t + c$, giving $v = 10\,e^{-0.25t}$

$v = 5$: $e^{-0.25t} = \frac{1}{2}$, so $t = \frac{\ln 2}{0.25}$

$s = \int v\,dt = -\frac{10}{0.25}e^{-0.25t} + c$; distance in this time $= \frac{10}{0.25}(1 - \frac{1}{2})$

Answer: (a) 2.77 s (b) 20 m

9 Points of inflection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the point of inflection of
$y = xe^{-x}$,
justifying that it is a point of inflection.

$dy/dx = e^{-x}(1 - x)$
$d^2y/dx^2 = e^{-x}(x - 2) = 0$, so $x = 2$
d^2y/dx^2 changes sign either side of $x = 2$
$y = 2e^{-2}$

Answer: $(2, 2e^{-2})$

10 Rejected batches: binomial inside binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box holds 11 components, each faulty with probability 0.1, independently. A box is rejected if it has at least 3 faulty components.

(a) Find the probability that a box is rejected.

(b) 12 boxes are inspected. Find the probability that at least 2 are rejected.

Let $X \sim B(11, 0.1)$. $P(\text{rejected}) = P(X \geq 3) = 1 - P(X \leq 2) = 0.08956$

Let Y = number of rejected boxes, $Y \sim B(12, 0.08956)$

$P(Y \geq 2) = 1 - P(Y \leq 1) = 0.293$

Answer: (a) 0.0896 (b) 0.293