Day 65 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Screening test and Bayes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A faulty component is present in 5% of a population. A test is positive for 95% of those with it and for 10% of those without it.

Find P(positive) and P(has it | positive).

P(positive) = 0.05 × 0.95 + 0.95 × 0.1

= 0.0475 + 0.095 = 0.1425

P(has it | positive) = $\frac{0.0475}{0.1425}$

= 0.3333

Answer: (a) 0.1425 (b) 0.333

2 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground with speed 63.7 m s^{-1} at an angle α above the horizontal, where $\tan \alpha = \frac{5}{12}$. Find (a) the greatest height, (b) the speed when it is at half of this height (3 s.f.). Take g = 9.8 m s^{-2}.

Vertical: $0 = 24.5^2 - 2 \times 9.8 \times H$, so H = 30.625

At height $\frac{H}{2}$ = 15.3125: $v_y^2 = 24.5^2 - 2 \times 9.8 \times 15.3125 = 300.125$

v_x = 58.8 (constant), so speed^2 = 3457.44 + 300.125 = 3757.565

Answer: (a) 30.625 m (b) 61.3 m s^{-1}

3 Normal test: critical region and Type II error (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ N(μ, 9). H_0: μ = 58 is tested against H_1: μ < 58 using the mean of a sample of size 25 at the 10% level.

(a) Find the critical region for the sample mean.

(b) Find the probability of a Type II error when μ = 57.5.

Under H_0, sample mean ~ N(58, $\frac{9}{25}$), standard error = 0.6000

z = 1.2816 for the lower tail: critical value = 58 − 1.2816 × 0.6000 = 57.23

Type II error: do not reject H_0, sample mean > 57.23, where mean ~ N(57.5, 0.6000)

$z = \frac{57.23 - 57.5}{0.6000} = -0.448$

Probability = 0.673

Answer: (a) sample mean < 57.2 (b) 0.673

4 Compound interest and savings (GP) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is dropped from a height of 4 m. After each bounce it rises to $\frac{3}{4}$ of its previous height.

(a) Find the total distance it travels before it stops.

(b) After how many bounces does it rise less than 0.01 m?

(a) Down H, then up and down each time: $H + 2Hr + 2Hr^2 + ...$

$= H + \frac{2Hr}{1 - r} = 4 + \frac{2 \times 4 \times \frac{3}{4}}{1 - \frac{3}{4}} = 28.$

(b) Rebound height after n bounces = $4(\frac{3}{4})^n < 0.01$.

$n > \frac{\log(0.0025)}{\log(0.75)} = 20.83$, so n = 21.

Answer: (a) 28 m (b) 21 bounces

5 Addition formulae with exact values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is acute and B is acute. $\cos A = \frac{20}{29}$ and $\cos B = \frac{15}{17}$.

Find the exact values of cos(A + B) and tan(A + B).

$\cos A = \frac{20}{29}$, $\sin A = \frac{21}{29}$; $\sin B = \frac{8}{17}$, $\cos B = \frac{15}{17}$

$\cos(A + B) = \cos A \cos B - \sin A \sin B = \frac{132}{493}$

$\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\frac{475}{493}}{\frac{132}{493}} = \frac{475}{132}$

Answer: $\cos(A + B) = \frac{132}{493}$, $\tan(A + B) = \frac{475}{132}$

6 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both

$x^2 + 4x + 3 > 0$ and $2x + 4 < 2$.

Quadratic: $(x + 3)(x + 1) > 0$ gives $x < -3$ or $x > -1$.
Linear: $2x + 4 < 2$ gives $x < -1$.
Both must hold: take the overlap of the two sets.
Answer: $x < -3$

7 Derivative of the inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve is given by $x = e^y + 4y$.

Find dy/dx and d^2y/dx^2 when $y = 0$.

$dx/dy = e^y + 4 = 5$ when $y = 0$, so $dy/dx = \frac{1}{dx/dy} = \frac{1}{5}$

$d^2x/dy^2 = 1$ when $y = 0$

$d^2y/dx^2 = -\frac{d^2x}{dy^2/(dx/dy)^3} = -\frac{1}{5^3}$

Answer: $dy/dx = \frac{1}{5}$, $d^2y/dx^2 = -\frac{1}{125}$

8 Tangent and axes triangle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tangent to $y = x\sqrt{3x + 6}$ at the point where $x = 1$
meets the axes at A and B. Find the exact area of triangle OAB,
where O is the origin.

At $x = 1$: $y = 3$. Differentiate: gradient $= \frac{7}{2}$

Tangent: $y - 3 = \frac{7}{2}(x - 1)$, so $y = \frac{7}{2}x - \frac{1}{2}$

y-intercept $-\frac{1}{2}$; x-intercept $\frac{1}{7}$

Area $= \frac{1}{2} \times |-\frac{1}{2}| \times |\frac{1}{7}|$

Answer: $\frac{1}{28}$

9 Block on a plank: limiting slip (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A plank of mass 5 kg lies on a smooth horizontal table, with a block of mass 3 kg on it ($\mu = 0.25$ between them). A horizontal force P N acts on the block.

(a) Find the greatest P for which the block does not slip. (b) Find the accelerations of block and plank when P = 35.28. Take $g = 9.8$ m s^{-2}.

No slipping: common acceleration $a = \frac{P}{8}$. The plank is accelerated only by friction: $F = 5a$

Limiting friction: $F = \mu mg = 0.25 \times 3 \times 9.8 = 7.35$, so $a = \frac{7.35}{5} = 1.47$

P_max = 8 × 1.47 = 11.76

P = 35.28 > 11.76, so the block slips. Plank: $a = \frac{7.35}{5} = 1.47$

Block: $35.28 - 7.35 = 3a$, so $a = 9.31$

Answer: (a) 11.76 N (b) block 9.31 m s^{-2}, plank 1.47 m s^{-2}

10 Binomial expansion (integer powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of $(3x + \frac{1}{x})^{10}$.

General term: $C(10,r)(3x)^{10-r}(\frac{1}{x})^{r}$.

Power of x: $1(10 - r) - 1r = 0$, so $r = 5$.

Term = $C(10,5) \times 3^5 \times (1)^5 = 61236$.

Answer: 61236