Day 61 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 One-tailed binomial test with data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A company claims that 25% of its customers renew their policy. A sample of 30 customers is taken, and 3 renew. Test, at the 1% significance level, the claim that the proportion is lower.
Give your conclusion in context.

H_0: p = 0.25, H_1: p < 0.25. Let X ~ B(30, 0.25)
P(X ≤ 3) = 0.0374
0.0374 is greater than 1%, so do not reject H_0
There is insufficient evidence that the proportion is lower than the claimed 0.25
Answer: Do not reject H_0: p-value 0.0374 > 1%

2 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ax^n$ passes through (1, 2) and (3, 54).
Find the values of a and n.

$2 = a \times 1^n$ and $54 = a \times 3^n$
Divide: $3^n = 27$ so n = 3
$a = 2 \div 1^3 = 2$
Answer: a = 2, n = 3

3 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = −2x − 1 and $g(x) = x^2 - 4$.
Solve fg(x) = gf(x).

$fg(x) = -2(x^2 - 4) - 1 = -2x^2 + 7$.
$gf(x) = (-2x - 1)^2 - 4 = 4x^2 + 4x - 3$.
Equate: $6x^2 + 4x - 10 = 0$.
Solve the quadratic: $x = -\frac{5}{3}$ or x = 1.

Answer: $x = -\frac{5}{3}$ or x = 1

4 Force applied then removed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 10 kg is at rest on rough horizontal ground (μ = 0.1). A horizontal force of 23.8 N acts on it for 7 s and is then removed.
Find (a) the speed when the force is removed, (b) the total distance travelled before the box stops. Take g = 9.8 m s^{-2}.

Friction = μmg = 0.1 × 10 × 9.8 = 9.8 N
While pushed: 23.8 − 9.8 = 10 a, so a = 1.4
v = 1.4 × 7 = 9.8; $s_1 = \frac{1}{2} \times 1.4 \times 7^2 = 34.3$
After release: deceleration = μg = 0.98, so $s_2 = \frac{9.8^2}{2 \times 0.98} = 49$
Total distance = 34.3 + 49
Answer: (a) 9.8 m s^{-1} (b) 83.3 m

5 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = \ln t$, $y = 5t^2 + 3t$, t > 0.
Find the equation of the tangent where t = 1, in the form y = mx + c.

$dx/dt = \frac{1}{t}$, dy/dt = 10t + 3
dy/dx = t(10t + 3) which is 13 at t = 1
At t = 1: x = 0, y = 8
Answer: y = 13x + 8

6 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (4 kg) lies on a rough table with $\mu = 0.1$, and is joined by a light string over a smooth pulley (see diagram) to particle B (2 kg) on a smooth slope, $\sin\alpha = \frac{4}{5}$.

Released from rest, the string is cut when B has moved 1 m. Find the further distance A slides before stopping.

A: $T - 0.1 \times 4 \times 9.8 = 4a$. B: $2 \times 9.8 \times \frac{4}{5} - T = 2a$

Adding: $9.8(2 \times \frac{4}{5} - 0.1 \times 4) = 6a$, so $a = 1.96$

$T = 4(1.96 + 0.1 \times 9.8) = 11.76$

When cut: $v^2 = 2 \times 1.96 \times 1 = 3.92$. Then A decelerates at $\mu g = 0.98$

Further distance $= \frac{3.92}{2 \times 0.98} = 2$

Answer: 2 m

7 Arithmetic series from a sum formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence has $u_3 = 4$ and $S_{12} = 216$.

(a) Find the first term and common difference.

(b) Find the least n for which $S_n > 611$.

u_3: $a + 2d = 4$. S_{12}: $12(2a + 11d)/2 = 216$, so $12a + 66d = 216$.

Solving: $a = -4$, $d = 4$.

$S_n = \frac{n}{2}(-8 + 4n - 4) = \frac{n(4n - 12)}{2}$.

Test: $S_{19} = 608 \leq 611$, $S_{20} = 680 > 611$.

Answer: (a) $a = -4$, $d = 4$ (b) $n = 20$

8 Normal: find mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The scores in an aptitude test are normally distributed. 15% are less than 38 and 15% are greater than 75. Find the mean and standard deviation, each correct to 3 significant figures.

Let $X \sim N(\mu, \sigma^2)$. $P(X < 38) = 0.15$ gives $\frac{38 - \mu}{\sigma} = -1.0364$

$P(X > 75) = 0.15$ gives $\frac{75 - \mu}{\sigma} = 1.0364$

Subtract: $\frac{37}{\sigma} = 1.0364 - (-1.0364) = 2.0729$

$\sigma = 17.85$

$\mu = 38 - (-1.0364)(17.85) = 56.50$

Answer: $\mu = 56.5$, $\sigma = 17.8$

9 Area under a parametric curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = t$, $y = te^{-t}$, for $0 \leq t \leq 1$.

Find the exact area under the curve between $t = 0$ and $t = 1$, above the x-axis.

$dx/dt = 1$, so area $= \int_0^1 te^{-t}\,dt$

Parts: $\int te^{-t}dt = -te^{-t} - e^{-t}$

$[-te^{-t} - e^{-t}]$ between 0 and 1 $= 1 - 2e^{-1}$

Answer: $1 - 2e^{-1}$

10 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sum_{r=1}^{n}(2r + 5) = 1591$, find n.

$\sum_{r=1}^{n}(2r + 5) = 2 \times \frac{n(n + 1)}{2} + 5n.$

$\frac{2\,n(n + 1)}{2} + 5n = 1591$, so $2n^2 + 12n - 3182 = 0$.

Solving $(n > 0)$: $n = 37$.

Answer: n = 37