Day 6 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 642 inclusive that are divisible by 3 or by 4, but not by both.

Multiples of 3: 214 terms (3 to 642): sum = 69015.
Multiples of 4: 160 terms: sum = 51520.
Multiples of both, i.e. of 12: 53 terms: sum = 17172.
Required sum = 69015 + 51520 − 2 × 17172.
Answer: 86191

2 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cot(2\theta - 30°) = \frac{1}{\sqrt{3}}$ for $-180° \le \theta \le 180°$.

Let $u = 2\theta - 30°$; then $\cot u = \frac{1}{\sqrt{3}}$
Base solutions: $u = 60°$ (+ multiples of 180°)
$\theta = (u + 30°) \div 2$, keeping those in range
Answer: θ = −-135°, −-45°, 45°, 135°

3 Velocity-time graph with a return (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the velocity-time graph of a particle P moving on a straight line (velocity in m s^{-1}, time in s). At time 9 s, P is back at its starting point.
Find (a) the value of W, (b) the greatest distance of P from its starting point.

Net area = 0 (back at start): ½ × 3 × 6 + ½(6 − W)(3) − W(3) = 0
9 + 9 − 1.5W − 3W = 0, so 4.5W = 18
W = 4

v = 0 where the line from (3, 6) to (6, −4) crosses the axis: $t = 3 + 3 \times \frac{6}{10} = 4.8$

Greatest distance = area of triangle = ½ × 4.8 × 6
Answer: (a) W = 4 (b) 14.4 m

4 Arithmetic series from a sum formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The sum of the first n terms of a sequence is $S_n = 3n^2 + 7n$.
(a) Find an expression for u_n.
(b) Find the least n for which $u_n > 268$.

$u_n = S_n - S_{n-1} = 3n^2 + 7n - [3(n - 1)^2 + 7(n - 1)]$
$= 6n + 4$.
$u_1 = S_1 = 10$ also fits, so $u_n = 6n + 4$ for all n.
$6n + 4 > 268 \Rightarrow n > 44$, so n = 45.
Answer: (a) u_n = 6n + 4 (b) n = 45

5 Large data set: linear change of units (AQA, Edexcel, OCR, MEI, Eduqas)

Large Data Set, daily mean air temperature (°C), Heathrow, July 2015 (n = 30): mean 18.9, sd 3. (F = 1.8C + 32)
(a) Find the mean and sd in °F.
(b) Is a reading of 23.4 more than 2 sd above the mean?

New value = 1.8 × old + 32, so mean = 1.8 × 18.9 + 32 = 66.02
The constant shifts the mean but not the spread; only the multiplier affects σ
σ = 1.8 × 3 = 5.400
Outlier cut-off = mean + 2σ = 18.9 + 6 = 24.9 (original units)
23.4 is less than 24.9, so no
Answer: (a) mean 66.0, sd 5.40 (b) no

6 Particle on a smooth slope joined to hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (5 kg) and B (5 kg) are joined by a light string over a smooth pulley (see diagram). A lies on a smooth slope, $\sin\alpha = \frac{3}{5}$, and B is 3 m above the ground.

Released from rest, find the further distance A moves up the slope after B lands (A does not reach the pulley).

A: $T - 5 \times 9.8 \times \frac{3}{5} = 5$ a. B: $5 \times 9.8 - T = 5$ a

Adding: $9.8(5 - 5 \times \frac{3}{5}) = 10$ a, so a = 1.96

$T = 5(9.8 - 1.96) = 39.2$

When B lands: $v^2 = 2 \times 1.96 \times 3 = 11.76$

Then A decelerates at g sin α = 5.88: further distance = $\frac{11.76}{2 \times 5.88} = 1$

Answer: 1 m

7 Indices: quadratic in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $5^{2x} - 123 \times 5^x - 250 = 0$.

Let $y = 5^x$: $y^2 - 123y - 250 = 0$.

$(y - 125)(y + 2) = 0$, so $y = 125$ or $y = -2$.

$5^x > 0$ for all x, so $y = -2$ is rejected.

$5^x = 125$ gives $x = 3$.

Answer: x = 3

8 Triangle in 3D: angle and area (CIE (9709); Further Maths on other boards)

The points A, B, C have coordinates (5, 5, 0), (5, 1, 2), (0, −1, −1).

Find angle ABC to 1 d.p., and the exact area of triangle ABC.

$\mathrm{BA} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix}$, $\mathrm{BC} = \begin{pmatrix} -5 \\ -2 \\ -3 \end{pmatrix}$; $\mathrm{BA} \cdot \mathrm{BC} = -2$

$|\mathrm{BA}| = \sqrt{20}$, $|\mathrm{BC}| = \sqrt{38}$

$\cos \mathrm{ABC} = \frac{-2}{\sqrt{20}\sqrt{38}} = -0.0725$, so ABC = 94.2°

$|\mathrm{BA} \times \mathrm{BC}| = \left|\begin{pmatrix} -16 \\ 10 \\ 20 \end{pmatrix}\right| = \sqrt{756}$

Area = $\frac{1}{2}|\mathrm{BA} \times \mathrm{BC}|$

Answer: ABC = 94.2°, area = $3\sqrt{21}$

9 Reading a log graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = ab^x$. The graph of lg y against x is the straight line shown.

Find a and b to 3 significant figures.

lg y = lg a + x lg b. The intercept is lg a = 0.5, so $a = 10^{0.5} = 3.162$

Gradient = $\frac{0.9 - 0.5}{4} = 0.1$

lg b = 0.1 so $b = 10^{0.1} = 1.259$

Answer: a = 3.16, b = 1.26

10 Area under a parametric curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 4t$, $y = te^{-t}$, for $0 \leq t \leq 1$.
Find the exact area under the curve between $t = 0$ and $t = 1$,
above the x-axis.

$dx/dt = 4$, so area $= 4\int_0^1 te^{-t}\,dt$

Parts: $\int te^{-t}dt = -te^{-t} - e^{-t}$

$[-te^{-t} - e^{-t}]$ between 0 and 1 $= 1 - 2e^{-1}$

Answer: $4 - 8e^{-1}$