**Day 59** **A Level Maths: S Tier 10-a-day: Model Solutions**

## 1 Compound change with logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A drug dose of 100 mg decays by 20% each hour.
Find the first whole number of hours after which less than 2 mg remains.

$100 \times 0.80^n < 2$, so $0.80^n < \frac{1}{50}$

$n > \frac{\ln \frac{1}{50}}{\ln 0.80} = 17.53$ ($\ln 0.80$ is negative, so the inequality reverses)

$n = 18$

**Answer: 18**

## 2 Parametric curve and line intersection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has parametric equations
$x = t$, $y = 3t^2$. The line $y = -15x - 18$
meets C at A and B. Find the exact length AB.

Substitute: $3t^2 = -15t - 18$.

$3t^2 + 15t + 18 = 0 \Rightarrow t = -2$ or $t = -3$.

A$(-2, 12)$, B$(-3, 27)$.

$AB^2 = 1^2 + (-15)^2 = 226$.

**Answer: AB** $= \sqrt{226}$

## 3 Resultant of forces: unknown constant and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(2\mathbf{i} - \mathbf{j})$ N, $(2\mathbf{i} + k\mathbf{j})$ N and $(-8\mathbf{i} - 4\mathbf{j})$ N act on a particle of mass 3 kg. The resultant is parallel to $(-\mathbf{i} - \mathbf{j})$.
Find (a) the value of k, (b) the magnitude of the acceleration of the particle, to 3 s.f.

Resultant $= (-4)\mathbf{i} + (-5 + k)\mathbf{j}$

Parallel to $(-\mathbf{i} - \mathbf{j})$: $\frac{-5 + k}{-4} = \frac{-1}{-1}$, so $k = 1$

$\mathbf{R} = (-4\mathbf{i} - 4\mathbf{j})$ N, so $|R| = \sqrt{(-4^2 + -4^2)} = 5.657$

$a = |R| \div 3$

**Answer: (a) k = 1 (b) 1.89 m s$^{-2}$**

## 4 sec, cosec and tan/cot quadratics (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3\sec^2\theta + 4\tan\theta - 2 = 0$
for $0 \leq \theta < 360°$, giving answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $3\tan^2\theta + 4\tan\theta + 1 = 0$

$(3\tan\theta + 1)(\tan\theta + 1) = 0$

$\tan\theta = -\frac{1}{3}$ or $\tan\theta = -1$

$\theta = 161.6°$ and $135.0°$ (+180°)

**Answer: $\theta$ = 135.0°, 161.6°, 315.0°, 341.6°**

## 5 Normal: unknown standard deviation then a second probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The lengths of a component, in mm are modelled by $N(82, \sigma^2)$. The probability of a value above 97 is 0.10.
(a) Find $\sigma$.
(b) Find the probability of a value below 98.

$P(X > 97) = 0.10$ so $Z = \frac{97 - 82}{\sigma} = 1.2816$

$\sigma = \frac{15}{1.2816} = 11.70$

$z = \frac{98 - 82}{11.70} = 1.367$

$P(X < 98) = P(Z < 1.367) = 0.914$

**Answer: (a) 11.7 (b) 0.914**

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## 6 Trapezium rule and over/under estimates (AQA, Edexcel, OCR, MEI, Eduqas (trapezium rule))

Use the trapezium rule with 5 strips to estimate

$\int_0^2 \frac{1}{\sqrt{1+x}}\,dx$, to 4 s.f.

Is it an overestimate or an underestimate? Explain.

$h = \frac{2}{5}$; ordinates at $x = 0, \frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \ldots, 2$

Sum $= \frac{h}{2}[y_0 + y_5 + 2(y_1 + \ldots + y_4)] = 1.469$

$f''(x) > 0$ on the interval, so the curve is convex and the trapezia lie above the curve

**Answer: 1.469; overestimate**

## 7 Rational inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{5}{(x-4)} > \frac{4}{(x-1)}$.

Multiply by the positive quantity $(x-4)^2(x-1)^2$:

$5(x-4)(x-1)^2 > 4(x-1)(x-4)^2$.

Move to one side and factorise: $(x-4)(x-1)(x+11) > 0$.

Critical values: 4, 1, −11. Test the four regions.

**Answer: $-11 < x < 1$ or $x > 4$**

## 8 Critical region, significance level and Type II error (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A test of $H_0$: $p = 0.5$ against $H_1$: $p > 0.5$ uses a sample of 50 at the 10% level.

(a) Find the critical region for X, the number of successes.

(b) Find the actual significance level.

(c) Find P(Type II error) when in fact $p = 0.7$.

$X \sim B(50, 0.5)$. $P(X \geq 31) = 0.0595$ and $P(X \geq 30) = 0.101$

The first is at most 0.1 and the second is not, so the critical region is $X \geq 31$

Actual significance level $= P(X \geq 31) = 0.0595$

Type II error: $H_0$ is not rejected, so $X \leq 30$, with $X \sim B(50, 0.7)$

$P(X \leq 30) = 0.0848$

**Answer: (a) $X \geq 31$ (b) 0.0595 (c) 0.0848**

## 9 Acceleration given, find when P returns to O (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time $t = 0$, P passes through the fixed point O with velocity 60 m $s^{-1}$. At time t seconds, P has acceleration $(3t - 23)$ m $s^{-2}$.

Find (a) the times at which P is at O again, (b) the greatest distance of P from O before it first returns to O.

$v = \int a\,dt = 1.5t^2 - 23t + 60$ ($v = 60$ when $t = 0$)

$s = \int v\,dt = 0.5t^3 - 11.5t^2 + 60t$ ($s = 0$ when $t = 0$)

$s = 0.5t(t-8)(t-15)$, so P is at O when $t = 8$ and $t = 15$

Greatest distance when $v = 0$: $1.5t^2 - 23t + 60 = 0$, so $t = \frac{10}{3}$ (the smaller root)

$|s| = 0.5 \times \frac{10}{3} \times \frac{14}{3} \times \frac{35}{3}$

**Answer: (a) $t = 8$, $t = 15$ (b) $\frac{2450}{27}$ (= 90.7) m**

## 10 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
$x = 3 \sin t$, $y = 2 \cos 2t$.
Find y in terms of x.

$\cos 2t = 1 - 2\sin^2 t$.

$\sin t = \frac{x}{3}$.

$y = 2(1 - 2 \times \frac{x^2}{9}) = 2 - \frac{4}{9} x^2$.

**Answer: $y = 2 - \frac{4}{9}x^2$**