**Day 55** **A Level Maths: S Tier 10-a-day: Model Solutions**

## 1 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cot(\theta - 10°) = \sqrt{3}$
for $0° \leq \theta \leq 360°$.

Let $u = \theta - 10°$; then $\cot u = \sqrt{3}$
Base solutions: $u = 30°$ (+ multiples of 180°)
$\theta = (u + 10°) \div 1$, keeping those in range
**Answer: $\theta = 40°, 220°$**

## 2 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^3 - x^2 - 16x + 16 < 0$.

Use the factor theorem to find roots: $-4, 1, 4$.
$f(x) = (x + 4)(x - 1)(x - 4)$.
Sign of f in each region (left to right): $-, +, -, +$.
Take the regions where $f < 0$ (include roots if $\leq$ or $\geq$).
**Answer: $x < -4$ or $1 < x < 4$**

## 3 R-form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Express $4\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$. Give $\alpha$ to 1 d.p.
(b) Hence solve $4\cos\theta - 3\sin\theta = 3$ for $0 \leq \theta < 360°$, giving answers to 1 d.p.

$R = \sqrt{4^2 + 3^2} = 5$ and $\tan\alpha = \frac{3}{4}$, so $\alpha = 36.9°$

So $4\cos\theta - 3\sin\theta \equiv 5\cos(\theta + 36.9°)$

$\cos(\theta + \alpha) = \frac{3}{5}$

$\theta + \alpha = \pm 53.1°$ (using $\alpha = 36.9°$)
Adding or subtracting multiples of 360° to give $0 \leq \theta < 360°$
**Answer: (a) $R = 5$, $\alpha = 36.9°$ (b) $\theta = 16.3°, 270.0°$**

## 4 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 16 and upper quartile 28. An outlier is a value more than 1 × IQR beyond the nearest quartile.
(a) Find the range of values that are not outliers.
(b) 8 observations have mean 16. The largest is 46, an outlier, and it is removed. Find the new mean.

IQR = 28 − 16 = 12, so 1 × IQR = 12
Lower limit = 16 − 12 = 4, upper limit = 28 + 12 = 40
$\Sigma x = 8 \times 16 = 128$, so without 46: $\Sigma x = 82$

New mean = $\frac{82}{7}$

**Answer: (a) 4 to 40 (b) 11.71**

## 5 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both
$x^2 - 10x + 21 < 0$ and $-x - 4 \leq -8$.

Quadratic: $(x - 3)(x - 7) < 0$ gives $3 < x < 7$.
Linear: $-x - 4 \leq -8$ gives $x \geq 4$. (the inequality reverses as we divide by a negative)
Both must hold: take the overlap of the two sets.
**Answer: $4 \leq x < 7$**

## 6 Integration by substitution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution u = ln x to find the exact value of

$\int_1^{e^3} \frac{(\ln x)^3}{x}\,dx$

$du = \frac{1}{x}\,dx$; limits $x = 1, e^3$ become $u = 0, 3$

Integral = $\int_0^3 u^3\,du$

Evaluate the limits

**Answer:** $\frac{81}{4}$

## 7 Just clearing a wall at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 45° to the horizontal. The ball just passes over the top of a vertical wall of height 25.2 m, at a horizontal distance of 90 m from O.

Find (a) the speed of projection, (b) the greatest height of the ball above the ground, to 3 s.f. Take g = 9.8 m $s^{-2}$.

At 45°: $x = \frac{u\,t}{\sqrt{2}}$ and $y = \frac{u\,t}{\sqrt{2}} - 4.9t^2$, so $y = x - \frac{9.8x^2}{u^2}$

$25.2 = 90 - \frac{9.8 \times 90^2}{u^2}$, so $u^2 = \frac{9.8 \times 8100}{64.8} = 1225$

u = 35

Greatest height = $\frac{(u \sin 45°)^2}{(2 \times 9.8)} = \frac{1225}{39.2}$

**Answer: (a) 35 m $s^{-1}$ (b) 31.2 m**

## 8 Rejected batches: binomial inside binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box holds 9 components, each faulty with probability 0.1, independently. A box is rejected if it has at least 3 faulty components.

(a) Find the probability that a box is rejected.

(b) 8 boxes are inspected. Find the probability that at least 2 are rejected.

Let X ~ B(9, 0.1). P(rejected) = $P(X \geq 3) = 1 - P(X \leq 2) = 0.05297$

Let Y = number of rejected boxes, Y ~ B(8, 0.05297)

$P(Y \geq 2) = 1 - P(Y \leq 1) = 0.0635$

**Answer: (a) 0.0530 (b) 0.0635**

## 9 Transformations of y = x^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe a sequence of two transformations that maps the curve $y = x^2$ onto the curve $y = 4x^2 - 16x + 23$.

Complete the square: $y = 4(x - 2)^2 + 7$.

Step 1: $y = x^2 \rightarrow y = 4x^2$: a stretch, scale factor 4, parallel to the y-axis.

Step 2: replace x by (x − 2) and add 7: translation by $\begin{pmatrix} 2 \\ 7 \end{pmatrix}$.

**Answer: a stretch, scale factor 4, parallel to the y-axis, then translation** $\begin{pmatrix} 2 \\ 7 \end{pmatrix}$

## 10 Hinged rod supported by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (3 m, 80 N) is hinged at A to a vertical wall and held horizontal by a string BC, C on the wall above A, with $\tan\alpha = \frac{5}{12}$, where $\alpha$ is the angle between string and rod. A 50 N particle hangs at P, AP = 1.5 m.

Find (a) the tension, (b) the magnitude and direction of the hinge force (3 s.f. and 1 d.p.).

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$. Moments about A: $T\sin\alpha \times 3 = 80 \times 1.5 + 50 \times 1.5$

$T \times \frac{5}{13} \times 3 = 195$, so $T = 169$

Horizontal: $H = T\cos\alpha = 156$. Vertical: $V + T\sin\alpha = 80 + 50$, so $V = 65$

Force $= \sqrt{(156^2 + 65^2)} = 169$, angle $= \tan^{-1}(\frac{65}{156}) = 22.6°$ above the horizontal

**Answer: (a) 169 N (b) 169 N at 22.6° above the horizontal (away from the wall)**