Day 50 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Two particles over a pulley: height reached (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 10 kg and 4 kg hang on a light string over a smooth fixed pulley. The 10 kg particle is 1.75 m above the ground; the 4 kg one is on the ground.

Released from rest, find (a) the tension, (b) the greatest height reached by the 4 kg particle. Take $g = 9.8$ m s^{-2}.

$10g - T = 10a$ and $T - 4g = 4a$, so $a = \frac{10-4}{10+4} \times 9.8 = 4.2$

$T = 10(9.8 - 4.2) = 56$

The 4 kg particle rises 1.75 m before the 10 kg particle lands, reaching $v^2 = 2 \times 4.2 \times 1.75 = 14.7$

The string goes slack, so it then moves freely under gravity: extra height $= \frac{v^2}{2g} = \frac{14.7}{19.6} = 0.75$

Greatest height $= 1.75 + 0.75$

Answer: (a) 56 N (b) 2.5 m

2 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|x + 4| > -2x - 4$.

Boundary values: $x + 4 = -2x - 4$ gives $x = -\frac{8}{3}$;

$x + 4 = -(-2x - 4)$ gives $x = 0$.

Also the corner of $|x + 4|$ is at $x = -4$.

Testing each region (check the graph of each side) gives the solution set.

Answer: $x > -\frac{8}{3}$

3 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass M kg, carrying a passenger of mass 75 kg, hangs from a vertical cable. The tension in the cable is 4255 N and the floor exerts a force of 555 N on the passenger.

Find (a) the acceleration of the lift, with its direction, (b) the value of M. Take $g = 9.8$ m s^{-2}.

Passenger (upwards positive): $555 - 75 \times 9.8 = 75a$, so $a = -2.4$

$a = 2.4$ m s^{-2}, so the acceleration is downwards

Lift and passenger: $4255 - (M + 75) \times 9.8 = (M + 75) \times -2.4$

$(M + 75)(9.8 - 2.4) = 4255$, so $M + 75 = \frac{4255}{7.4} = 575$

Answer: (a) 2.4 m s^{-2} downwards (b) M = 500

4 Related rates of change (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cone with its vertex downwards and semi-vertical angle α, where $\tan\alpha = \frac{1}{2}$, is filled at 15π cm^{3}s^{-1}. Find the rate at which the depth h is rising when h = 2 cm. ($V = \frac{1}{3}\pi r^2 h$)

$r = \frac{h}{2}$, so $V = \frac{1}{3}\pi \times \frac{h^2}{4} \times h = \frac{\pi}{12}h^3$

$dV/dt = \frac{\pi}{4}h^2\, dh/dt$, so $15\pi = \frac{\pi}{4} \times 4 \times dh/dt$

$dh/dt = \frac{60}{4}$

Answer: 15 cm s^{-1}

5 Coded data: mean, standard deviation, Σx^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 25 observations x, $\Sigma(x - 20) = 10$ and $\Sigma(x - 20)^2 = 104$.
(a) Find the mean of x.
(b) Find the standard deviation of x.
(c) Find the value of Σx^2.

Mean of $(x - 20) = \frac{10}{25} = 0.4$, so mean of $x = 20.4$

Variance $= \frac{104}{25} - (0.4)^2 = 4$

$\sigma = \sqrt{4}$

$\Sigma x^2 = \Sigma(x - 20)^2 + 40\,\Sigma(x - 20) + 25 \times 20^2$

$= 104 + 400 + 10000$

Answer: (a) 20.4 (b) 2.00 (c) 10504

6 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 - xy + 2y^2 = 7$.
Find the coordinates of the points on C where the tangent is parallel to the x-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + -y}{-x + 4y}$

Horizontal tangent when $2x + -y = 0$, so $y = \frac{2x}{1}$

Substitute into the equation of C: $x^2 + (-xy) + (2y^2) = 7$

Solve for the remaining coordinate

Answer: (−1, −2) and (1, 2)

7 Transformations of y = x^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe a sequence of two transformations that maps the curve $y = x^2$ onto the curve $y = -3x^2 + 6x - 9$.

Complete the square: $y = -3(x - 1)^2 - 6$.

Step 1: $y = x^2 \rightarrow y = -3x^2$: a stretch, scale factor 3, parallel to the y-axis and a reflection in the x-axis.

Step 2: replace x by $(x - 1)$ and add -6: translation by $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$.

Answer: a stretch, scale factor 3, parallel to the y-axis and a reflection in the x-axis, then translation $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$

8 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 238 inclusive that are divisible by 2 or by 9, but not by both.

Multiples of 2: 119 terms (2 to 238): sum = 14280.
Multiples of 9: 26 terms: sum = 3159.
Multiples of both, i.e. of 18: 13 terms: sum = 1638.
Required sum = $14280 + 3159 - 2 \times 1638$.

Answer: 14163

9 Triangle in 3D: angle and area (CIE (9709); Further Maths on other boards)

The points A, B, C have coordinates (4, 2, 4), (−1, 0, 4), (1, 2, 3).
Find angle ABC to 1 d.p., and the exact area of triangle ABC.

$\text{BA} = \begin{pmatrix}5\\2\\0\end{pmatrix}$, $\text{BC} = \begin{pmatrix}2\\2\\-1\end{pmatrix}$; $\text{BA} \cdot \text{BC} = 14$

$|\text{BA}| = \sqrt{29}$, $|\text{BC}| = \sqrt{9}$

$\cos \text{ABC} = \frac{14}{\sqrt{29}\sqrt{9}} = 0.8666$, so $\text{ABC} = 29.9°$

$|\text{BA} \times \text{BC}| = \left|\begin{pmatrix}-2\\5\\6\end{pmatrix}\right| = \sqrt{65}$

$\text{Area} = \frac{1}{2}|\text{BA} \times \text{BC}|$

Answer: ABC = 29.9°, area = $\frac{1}{2}\sqrt{65}$

10 Reading a log graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = ab^x$. The graph of lg y against x is the straight line shown.
Find a and b to 3 significant figures.

$\lg y = \lg a + x \lg b$. The intercept is $\lg a = 1$, so $a = 10^1 = 10.00$

$\text{Gradient} = \frac{2.2 - 1}{4} = 0.3$

$\lg b = 0.3$ so $b = 10^{0.3} = 1.995$

Answer: a = 10.0, b = 2.00