Day 5	A Level Maths: S Tier 10-a-day
1 A ball is projected from the top of a vertical cliff, 58.8 m high, at 24.5 m s^{-1} at angle α above the horizontal ($\tan\alpha = \frac{4}{3}$), and lands on horizontal ground. Find (a) the time of flight and the distance from the cliff, (b) the speed and direction on landing. Take g = 9.8 m s^{-2}.	
2 A radioactive sample has mass $m = m_0e^{-kt}$ grams, t in days. Initially m = 160 and after 4 days m = 80. Find k, and the time for the mass to fall to 20 grams, to 3 s.f.	
3 A javelin is projected from a point O on horizontal ground with speed 24.5 m s^{-1} at an angle θ above the horizontal, where $\tan\theta = \frac{3}{4}$. Find (a) the greatest height, (b) the speed when it is at half of this height (3 s.f.). Take g = 9.8 m s^{-2}.	
4 Prove that $n^3 + 8n$ is divisible by 3 for every positive integer n.	
5 Find the coordinates of the stationary points of $y = 3\cos 2x + 6\cos x$ for $0 < x < 2\pi$, and determine their nature.	

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6 $f(x) = x^3 + ax^2 + bx + 60$, where a and b are constants. $(x - 3)$ is a factor of f(x), and f(x) leaves remainder 18 when divided by $(x - 2)$. Find a and b, and hence solve $f(x) = 0$.	
7 Each seed is faulty with probability 0.03, independently. Find the least number n of seeds that must be tested so that the probability of finding at least one faulty is greater than 0.99.	
8 A curve has parametric equations $x = 4 \sin t$, $y = 1 \cos 2t$. Find y in terms of x.	
9 The line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$. The point P has coordinates $(6, 3, -5)$. Find the coordinates of the foot F of the perpendicular from P to l, and the exact distance PF.	
10 A test of H_0: $p = 0.4$ against H_1: $p > 0.4$ uses a sample of 40 at the 5% level. (a) Find the critical region for X, the number of successes. (b) Find the actual significance level. (c) Find P(Type II error) when in fact $p = 0.5$.	