**Day 46** **A Level Maths: S Tier 10-a-day: Model Solutions**

## 1 Quadratic sequence from recurrence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$u_{n+1} = u_n + 2n + 2$, $u_1 = -5$.

(a) Show that $u_n = an^2 + bn + c$ and find a, b, c.

(b) Find the least n for which $u_n > 295$.

$$u_n = u_1 + \sum_{k=1}^{n-1}(2k + 2) = -5 + 2 \times \frac{(n-1)n}{2} + 2(n-1).$$

$= n^2 + n - 7$.

So $a = 1$, $b = 1$, $c = -7$.

$u_{16} = 265 \leq 295$ and $u_{17} = 299 > 295$.

**Answer: (a) a = 1, b = 1, c = −7 (b) n = 17**

## 2 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ab^x$ passes through (2, 50) and (5, 6250).

Find the exact values of a and b, and the value of y when x = 7.

$ab^2 = 50$ and $ab^5 = 6250$

Divide: $b^3 = 125$ so $b = 5$

$a = 50 \div 5^2 = 2$

$y = 2 \times 5^7$

**Answer: a = 2, b = 5, y = 156250**

## 3 Coded data: mean, standard deviation, Σx² (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 25 observations x, $\Sigma(x - 30) = 45$ and $\Sigma(x - 30)^2 = 406$.

(a) Find the mean of x.

(b) Find the standard deviation of x.

(c) Find the value of $\Sigma x^2$.

Mean of $(x - 30) = \frac{45}{25} = 1.8$, so mean of $x = 31.8$

Variance $= \frac{406}{25} - (1.8)^2 = 13$

$\sigma = \sqrt{13}$

$\Sigma x^2 = \Sigma(x - 30)^2 + 60\,\Sigma(x - 30) + 25 \times 30^2$

$= 406 + 2700 + 22500$

**Answer: (a) 31.8 (b) 3.61 (c) 25606**

## 4 Overtaking with a delayed start (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car P passes a point O travelling at a constant speed of 9.6 m $s^{-1}$. 2 s later, car Q leaves O from rest with constant acceleration 3 m $s^{-2}$, travelling in the same direction.

Q overtakes P at the point X. Find (a) the time Q takes to reach X, (b) the distance OX, (c) the speed of Q at X.

Let Q travel for t s. P: distance $= 9.6(t + 2)$; Q: distance $= \frac{1}{2} \times 3 \times t^2$

Equate: $3t^2 - 19.2t - 38.4 = 0$, so $(t - 8)(3t + 4.8) = 0$

$t = 8$ (the other root is negative)

OX $= 3 \times 8^2 / 2 = 96$ m, and speed $= 3 \times 8 = 24$

**Answer: (a) 8 s (b) 96 m (c) 24 m $s^{-1}$**

## 5 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows y = f(x), made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of

$y = -f(2x - 8)$.

Inside f: replace x by $2x - 8$, so x-coordinates become $(x + 8) \div 2$.

Outside: y-coordinates become $-1 \times y$.

A: (1, 3) B: (2, 4)

C: (6, −1) D: (7, −5)

**Answer: A′(1, 3), B′(2, 4), C′(6, −1), D′(7, −5)**

## 6 Fixed point iteration and rearrangement (AQA, Edexcel, OCR, MEI, Eduqas, CIE (iterative methods))

The equation $x^3 = 3x + 17$ has a root $\alpha$.
(a) Show that the equation can be written as $x = \sqrt[3]{(3x + 17)}$.
(b) Use $x_{n+1} = \sqrt[3]{(3x_n + 17)}$ with $x_0 = 3$ to find $x_1$, $x_2$, $x_3$
to 4 d.p., and hence state $\alpha$ to 3 d.p.

Take the cube root of both sides to obtain $x = \sqrt[3]{(3x + 17)}$
$x_1 = 2.9625$, $x_2 = 2.9582$, $x_3 = 2.9577$
The iteration converges (successive values agree to 3 d.p.): $\alpha = 2.958$
**Answer: $x_1 = 2.9625$, $x_2 = 2.9582$, $x_3 = 2.9577$; $\alpha = 2.958$**

## 7 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 284
inclusive that are divisible by 4 or by 6, but not by both.

Multiples of 4: 71 terms (4 to 284): sum = 10224.
Multiples of 6: 47 terms: sum = 6768.
Multiples of both, i.e. of 12: 23 terms: sum = 3312.
Required sum = 10224 + 6768 − 2 × 3312.
**Answer: 10368**

## 8 Conditional probability from given values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, P(A) = 0.75, P(B) = 0.6 and P(A | B) = 0.65.
(a) Find P(A ∩ B).
(b) Find P(A | B′).

$P(A \cap B) = P(A \mid B) \times P(B) = 0.65 \times 0.6 = 0.39$
$P(A \cup B) = 0.75 + 0.6 - 0.39 = 0.96$
$P(B') = 0.4$
$P(A \cap B') = P(A) - P(A \cap B) = 0.36$
$P(A \mid B') = \frac{0.36}{0.4}$

**Answer: (a) 0.39 (b) $\frac{9}{10}$**

## 9 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 + xy + 3y^2 = 132$.
Find the coordinates of the points on C where the
tangent is parallel to the y-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + y}{x + 6y}$

Vertical tangent when $x + 6y = 0$, so $x = -\frac{6y}{1}$

Substitute into the equation of C: $x^2 + (xy) + (3y^2) = 132$
Solve for the remaining coordinate
**Answer: (−12, 2) and (12, −2)**

## 10 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (2 kg) lies on a rough table with $\mu = 0.1$, and is joined by a light string over a smooth pulley (see diagram) to particle B (7 kg) on a smooth slope, $\sin\alpha = \frac{4}{5}$.

Released from rest, the string is cut when B has moved 4 m. Find the further distance A slides before stopping.

A: $T - 0.1 \times 2 \times 9.8 = 2a$. B: $7 \times 9.8 \times \frac{4}{5} - T = 7a$

Adding: $9.8(7 \times \frac{4}{5} - 0.1 \times 2) = 9a$, so $a = 5.88$

$T = 2(5.88 + 0.1 \times 9.8) = 13.72$

When cut: $v^2 = 2 \times 5.88 \times 4 = 47.04$. Then A decelerates at $\mu g = 0.98$

Further distance $= \frac{47.04}{2 \times 0.98} = 24$

**Answer: 24 m**