Day 44 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Shaded normal curve and the standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The pdf of $X \sim N(33, \sigma^2)$ is shown; the shaded area is symmetrical.
(a) Find σ. (b) Find $P(X > 48)$.

The area 0.8 leaves 0.200 in total in the tails, 0.100 in each

$P(X < 23) = 0.100$ so $\frac{23 - 33}{\sigma} = -1.2816$

$\sigma = \frac{10}{1.2816} = 7.803$

$P(X > 48) = P(Z > 1.922) = 0.0273$

Answer: (a) 7.80 (b) 0.0273

2 Geometric series: sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has sum to infinity 50 and second term 12. Find the possible values of the common ratio r and the first term a in each case.

$a = \frac{12}{r}$. $S_\infty = \frac{a}{1 - r} = \frac{12}{r(1 - r)} = 50$.

$50r^2 - 50r + 12 = 0$.

$(5r - 3)(5r - 2) = 0$: $r = \frac{3}{5}$ or $r = \frac{2}{5}$.

Need $|r| < 1$: $\frac{3}{5}$, $\frac{2}{5}$ valid.

Answer: $r = \frac{3}{5}$, $a = 20$ or $r = \frac{2}{5}$, $a = 30$

3 Differentiating a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 2^x - 5x$ has one stationary point.
Find its coordinates to 3 s.f. and show that it is a minimum.

$dy/dx = 2^x \ln 2 - 5 = 0$

$2^x = \frac{5}{\ln 2}$, $x = \frac{1}{\ln 2} \ln(\frac{5}{\ln 2}) = 2.85$

$d^2y/dx^2 = 2^x(\ln 2)^2 > 0$ for all x: minimum

$y = \frac{5}{\ln 2} - 5 \times x = -7.04$

Answer: (2.85, −7.04)

4 Cubic graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the cubic $y = f(x)$, which touches the x-axis once. Find f(x), giving your answer in expanded form.

Crosses at $x = -3$ and touches at $x = 4$, so $f(x) = k(x + 3)(x - 4)^2$.

At (0, 48): $48 = k(3)(-4)^2 = 48k$, so $k = 1$.

$f(x) = (x + 3)(x - 4)^2$, then expand.

Answer: $f(x) = x^3 - 5x^2 - 8x + 48$

5 Slide down a rough slope then along rough ground (CIE)

A particle starts from rest at the top of a rough slope of length 12 m ($\sin\alpha = \frac{4}{5}$, $\mu = 0.15$), then slides along rough horizontal ground with the same μ.
Use energy to find (a) its speed at the foot of the slope (3 s.f.), (b) the distance it slides on the ground. Take $g = 9.8$ m s^{-2}.

On the slope: $\frac{1}{2} m v^2 = m g \times 12 \sin\alpha - \mu m g \cos\alpha \times 12$

$v^2 = 2 \times 9.8 \times 12(\frac{4}{5} - 0.15 \times \frac{3}{5}) = 166.992$, so $v = 12.92$

On the ground: $\frac{1}{2} m v^2 = \mu m g x$, so $x = \frac{v^2}{2 \mu g} = \frac{166.992}{2.94} = 56.8$

Answer: (a) 12.9 m s^{-1} (b) 56.8 m

6 Most likely value of a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(12, \frac{3}{10})$.

(a) Show that $P(X = r + 1) \div P(X = r) = \frac{12 - r}{r + 1} \times \frac{3}{7}$.

(b) Find the most likely value of X and its probability (3 s.f.).

$P(X = r + 1) \div P(X = r) = \frac{C(12, r + 1)}{C(12, r)} \times \frac{p}{q} = \frac{12 - r}{r + 1} \times \frac{p}{q}$

With $p = \frac{3}{10}$ and $q = \frac{7}{10}$, $p \div q = \frac{3}{7}$

The ratio exceeds 1 when $(12 - r)\frac{3}{10} > (r + 1)\frac{7}{10}$, that is $r < 2.9$

So probabilities rise up to r = 3 and then fall; the mode is 3

P(X = 3) = 0.240

Answer: 3 with probability 0.240

7 Fixed point iteration and rearrangement (AQA, Edexcel, OCR, MEI, Eduqas, CIE (iterative methods))

The equation $x^3 = 3x + 4$ has a root α.

(a) Show that the equation can be written as $x = \sqrt[3]{(3x + 4)}$.

(b) Use $x_{n+1} = \sqrt[3]{(3x_n + 4)}$ with $x_0 = 3$ to find x_1, x_2, x_3 to 4 d.p., and hence state α to 3 d.p.

Take the cube root of both sides to obtain $x = \sqrt[3]{(3x + 4)}$

$x_1 = 2.3513$, $x_2 = 2.2276$, $x_3 = 2.2024$

The iteration converges (successive values agree to 3 d.p.): $\alpha = 2.196$

Answer: $x_1 = 2.3513$, $x_2 = 2.2276$, $x_3 = 2.2024$; $\alpha = 2.196$

8 Volumes of revolution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region bounded by $y = \frac{6}{x + 1}$, the x-axis and the lines x = 0 and x = 3 is rotated through 2π about the x-axis.

Find the exact volume of the solid formed.

$V = \pi\int_0^3 y^2\,dx = \pi\int_0^3 (\frac{6}{x + 1})^2\,dx$

Simplify the integrand, then integrate

$V = \pi \times (27)$

Answer: 27π

9 Binomial expansion (fractional or negative powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the binomial expansion of $(27 + 2x)^{\frac{2}{3}}$ in ascending powers of x up to and including the term in x^3.

State the values of x for which it is valid.

$(27 + 2x)^{\frac{2}{3}} = 27^{\frac{2}{3}}(1 + \frac{2}{27}x)^{\frac{2}{3}} = 9(1 + u)^{\frac{2}{3}}$, $u = \frac{2}{27}x$.

$(1 + u)^{\frac{2}{3}} = 1 + \frac{2}{3}u + (-\frac{1}{9})u^2 + \frac{4}{81}u^3 + \ldots$

Substitute u and multiply by 9.

Valid when $|u| < 1$, i.e. $|x| < \frac{27}{2}$.

Answer: $9 + \frac{4}{9}x - \frac{4}{729}x^2 + \frac{32}{177147}x^3 + \ldots$; $|x| < \frac{27}{2}$

10 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from level ground with horizontal and vertical velocity components 14.7 and 53.9 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take g = 9.8 m s^{-2}.

At 45° to the horizontal, |v_y| = v_x = 14.7
Moving upwards: 53.9 − 9.8t = 14.7, so t = 4
Moving downwards: 53.9 − 9.8t = −14.7, so t = 7
Horizontal distance = 14.7 × (7 − 4)
Answer: (a) 4 s and 7 s (b) 44.1 m