Day 41 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas, CIE (small angle))

θ is small and measured in radians. Use small angle approximations to find the value that $\frac{1 - \cos 5\theta}{\theta \tan 5\theta}$ approaches as $\theta \to 0$.

$1 - \cos 5\theta \approx \frac{1}{2}(5\theta)^2 = \frac{25}{2}\theta^2$, $\tan 5\theta \approx 5\theta$

Expression $\approx \frac{\frac{25}{2}\theta^2}{\theta \times 5\theta} = \frac{5}{2}$

Answer: $\frac{5}{2}$

2 Shaded normal curve and the standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The pdf of $X \sim N(83, \sigma^2)$ is shown; the shaded area is symmetrical.
(a) Find σ. (b) Find $P(X > 99)$.

The area 0.6 leaves 0.400 in total in the tails, 0.200 in each

$P(X < 73) = 0.200$ so $\frac{73 - 83}{\sigma} = -0.8416$

$\sigma = \frac{10}{0.8416} = 11.88$

$P(X > 99) = P(Z > 1.347) = 0.0891$

Answer: (a) 11.9 (b) 0.0891

3 Remainder theorem (quadratic divisor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when x^7 is divided by $(x + 1)(x + 2)$.

$x^7 = (x + 1)(x + 2)Q(x) + px + q$.
$x = -1$: $-1 = -p + q$.
$x = -2$: $-128 = -2p + q$.
Subtract: $p = 127$, so $p = 127$ and $q = 126$.
Answer: 127x + 126

4 Addition formulae with exact values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is acute and B is obtuse. $\sin A = \frac{24}{25}$ and $\cos B = -\frac{15}{17}$.

Find the exact values of $\sin(A + B)$ and $\cos(A - B)$.

$\cos A = \frac{7}{25}$, $\sin A = \frac{24}{25}$; $\sin B = \frac{8}{17}$, $\cos B = -\frac{15}{17}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B = -\frac{304}{425}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B = \frac{87}{425}$

Answer: $\sin(A + B) = -\frac{304}{425}$, $\cos(A - B) = \frac{87}{425}$

5 Particle in equilibrium on two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P of weight 30 N hangs in equilibrium from two light strings AP and BP attached to a horizontal ceiling at A and B (see diagram), where $\tan\alpha = \frac{3}{4}$ and $\tan\beta = \frac{4}{3}$.

Find (a) the tension in each string, (b) the greatest weight P could have if neither tension may exceed 68 N.

Horizontally: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_1 \times \frac{4}{5} = T_2 \times \frac{3}{5}$

Vertically: $T_1 \sin\alpha + T_2 \sin\beta = 30$, and $T_2 = \frac{4}{3} T_1$

$T_1(\frac{3}{5} + \frac{4}{3} \times \frac{4}{5}) = 30$, so $T_1 = 18$ and $T_2 = 24$

The tension in BP is the larger: it is 24 when W = 30

Tensions are proportional to W, so $W_{max} = 30 \times \frac{68}{24} = 85$

Answer: (a) AP: 18 N, BP: 24 N (b) 85 N

6 Transformations of y = x^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe a sequence of two transformations that maps the curve $y = x^2$ onto the curve $y = -x^2 + 6x - 3$.

Complete the square: $y = -(x - 3)^2 + 6$.

Step 1: $y = x^2 \rightarrow y = -1x^2$: a reflection in the x-axis.

Step 2: replace x by $(x - 3)$ and add 6: translation by $\begin{pmatrix}3\\6\end{pmatrix}$.

Answer: a reflection in the x-axis, then translation $\begin{pmatrix}\mathbf{3}\\\mathbf{6}\end{pmatrix}$

7 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = t^2$, $y = 2t^3 - 36t$.
Find the coordinates of the points on the curve where the gradient is -3.

$dx/dt = 2t$, $dy/dt = 6t^2 - 36$, so $dy/dx = \frac{6t^2 - 36}{2t}$

Set equal to -3: $6t^2 + 6t - 36 = 0$

$t = -3$ or $t = 2$

Answer: (9, 54) and (4, −56)

8 Screening test and Bayes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A rare allele is present in 5% of a population. A test is positive for 90% of those with it and for 5% of those without it. Find P(positive) and P(has it | positive).

P(positive) = 0.05 × 0.9 + 0.95 × 0.05

= 0.045 + 0.0475 = 0.0925

P(has it | positive) = $\frac{0.045}{0.0925}$

= 0.4865

Answer: (a) 0.0925 (b) 0.486

9 Perpendicular lines and area (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the diagram, line l_1 passes through A and B.
Line l_2 is perpendicular to l_1 at B and meets the
x-axis at C. Find the coordinates of C and the area of triangle ABC.

Gradient of $l_1 = \frac{2}{-4} = -\frac{1}{2}$.
Gradient of $l_2 = 2$. Equation: $y - 2 = 2x$.
$y = 0$: $x = -1$, so C$(-1, 0)$.
AB $= 2\sqrt{5}$, BC $= \sqrt{5}$: area = ½ × AB × BC.
Answer: C$(-1, 0)$; area 5

10 Block on a plank: limiting slip (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A plank of mass 8 kg lies on a smooth horizontal table, with a block of mass 4 kg on it ($\mu = 0.4$ between them). A horizontal force P N acts on the block.
(a) Find the greatest P for which the block does not slip. (b) Find the accelerations of block and plank when P = 29.4.
Take $g = 9.8$ m s^{-2}.

No slipping: common acceleration $a = \frac{P}{12}$. The plank is accelerated only by friction: $F = 8a$
Limiting friction: $F = \mu mg = 0.4 \times 4 \times 9.8 = 15.68$, so $a = \frac{15.68}{8} = 1.96$
P_max = 12 × 1.96 = 23.52
P = 29.4 > 23.52, so the block slips. Plank: $a = \frac{15.68}{8} = 1.96$
Block: $29.4 - 15.68 = 4a$, so $a = 3.43$
Answer: (a) 23.52 N (b) block 3.43 m s^{-2}, plank 1.96 m s^{-2}