Day 39 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(5, −5) and B(9, −3). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c - 5)^2 + 25 = (c - 9)^2 + 9$.
Expand and solve: 8c = 40, so c = 5.
Area = ½$|(x_A - c)y_B - (x_B - c)y_A|$
= ½|0 × (−3) − 4 × (−5)| = 10.
Answer: C(5, 0); area 10

2 Exponential growth and decay models (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A radioactive sample has mass $m = m_0e^{-kt}$ grams, t in days.

Initially m = 200 and after 5 days $m = \frac{200}{3}$.

Find k, and the time for the mass to fall to 20 grams, to 3 s.f.

$m_0 = 200$. $200\ e^{-5k} = \frac{200}{3}$ so $e^{-5k} = \frac{1}{3}$

$k = \frac{1}{5} \ln 3 = 0.220$

$200\ e^{-kt} = 20$ so $e^{kt} = 10$, $t = \frac{1}{k} \ln 10$

t = 10.5 days
Answer: k = 0.220, t = 10.5 days

3 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve |−3x + 1| < |−2x + 4|.

Square both sides: $(-3x + 1)^2 < (-2x + 4)^2$.
(−x − 3)(−5x + 5) < 0.
Critical values x = −3 and x = 1.
The quadratic has positive coefficient of x^2, so choose the inner region.
Answer: −3 < x < 1

4 Exponential times trigonometric (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = e^{2x} \cos 3x$. Show that $dy/dx = e^{2x}(2 \cos 3x - 3 \sin 3x)$.
Find the x coordinate of the stationary point with the smallest positive x (3 s.f.), and its nature.

Product rule: $dy/dx = (2)e^{2x} \cos 3x - e^{2x} \times 3 \sin 3x$
dy/dx = 0 requires 2 cos 3x = 3 sin 3x
$\tan 3x = \frac{2}{3}$, smallest positive 3x = 0.5880
x = 0.196
Second derivative test (or gradient change): a maximum
Answer: x = 0.196, maximum

5 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation $y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes x = 2 and y = 2, so p = −2 and q = 2.

$y = \frac{a}{x - 2} + 2$.

P(4, 4): $4 = \frac{a}{2} + 2$, so a = 4.

Answer: a = 4, p = −2, q = 2

6 Large data set: linear change of units (AQA, Edexcel, OCR, MEI, Eduqas)

Large Data Set, daily total sunshine (hours), Leuchars, July 2015 (n = 92): mean 7.3, sd 3.5. (1 hour = 60 minutes)
(a) Find the mean and sd in minutes.
(b) Is a reading of 15.7 more than 2 sd above the mean?

New value = 60 × old, so mean = 60 × 7.3 = 438.0
The constant shifts the mean but not the spread; only the multiplier affects σ
σ = 60 × 3.5 = 210.0
Outlier cut-off = mean + 2σ = 7.3 + 7 = 14.3 (original units)
15.7 is greater than 14.3, so yes
Answer: (a) mean 438, sd 210 (b) yes

7 Newton cooling model (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A drink cools according to $\theta = 25 + Ae^{-kt}$, where θ °C is the temperature after t minutes. Initially θ = 85, and after 4 minutes θ = 61.
Find A and k, then find when θ = 29 (3 s.f.).

t = 0: 25 + A = 85 so A = 60

t = 4: $25 + 60e^{-4k} = 61$ so $e^{-4k} = \frac{3}{5}$

$k = \frac{1}{4} \ln \frac{5}{3} = 0.128$

$60\, e^{-kt} = 4$ so $t = \frac{1}{k} \ln 15$

t = 21.2 minutes
Answer: A = 60, k = 0.128, t = 21.2 minutes

8 Ladder against a smooth wall: how high can a person climb (AQA, Edexcel, OCR, MEI)

A uniform ladder AB of length 5 m and weight 100 N has A on rough ground (μ = 0.25) and B against a smooth vertical wall. It makes angle θ with the ground, tan θ = 2.5.
A man of weight 500 N climbs the ladder. How far along the ladder can he go before it slips?

Wall reaction S horizontal, ground reaction R vertical, friction F = S. Moments about A: $S \times L \sin\theta = W \times \frac{L}{2} \cos\theta$
So $S = \frac{W}{2\tan\theta}$. Unloaded, R = W: need $\frac{W}{2\tan\theta} \le \mu W$, so $\mu \ge \frac{1}{2 \times 2.5} = 0.2$
With the man at distance x: R = 600, and moments about A give $S L \sin\theta = (100 \times 2.5 + 500x)\cos\theta$
Limiting: S = μR = 0.25 × 600 = 150, so 150 × 5 × 2.5 = 100 × 2.5 + 500x
500x = 1875 − 250, so x = 3.25
Answer: 3.25 m

9 Critical region, significance level and Type II error (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A test of H_0: p = 0.5 against H_1: p > 0.5 uses a sample of 50 at the 10% level.
(a) Find the critical region for X, the number of successes.
(b) Find the actual significance level.
(c) Find P(Type II error) when in fact p = 0.65.

X ~ B(50, 0.5). P(X ≥ 31) = 0.0595 and P(X ≥ 30) = 0.101
The first is at most 0.1 and the second is not, so the critical region is X ≥ 31
Actual significance level = P(X ≥ 31) = 0.0595
Type II error: H_0 is not rejected, so X ≤ 30, with X ~ B(50, 0.65)
P(X ≤ 30) = 0.274
Answer: (a) X ≥ 31 (b) 0.0595 (c) 0.274

10 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from level ground with horizontal and vertical velocity components 19.6 and 34.3 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take g = 9.8 m s^{-2}.

At 45° to the horizontal, |v_y| = v_x = 19.6
Moving upwards: 34.3 − 9.8t = 19.6, so t = 1.5
Moving downwards: 34.3 − 9.8t = −19.6, so t = 5.5
Horizontal distance = 19.6 × (5.5 − 1.5)
Answer: (a) 1.5 s and 5.5 s (b) 78.4 m