Day 365 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(2, −5) and B(−2, −3). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c-2)^2 + 25 = (c-(-2))^2 + 9$.
Expand and solve: $-8c = -16$, so c = 2.
Area = $\frac{1}{2}|(x_A - c)y_B - (x_B - c)y_A|$
$= \frac{1}{2}|0 \times (-3) - (-4) \times (-5)| = 10$.
Answer: C(2, 0); area 10

2 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves with constant acceleration. At t = 0 seconds P is at $(2\mathbf{i} - \mathbf{j})$ m with velocity $(-6\mathbf{i} - 14\mathbf{j})$ m s^{-1}. At t = 8, P has velocity $(10\mathbf{i} + 10\mathbf{j})$ m s^{-1}.
Find (a) the acceleration of P, (b) the time at which P moves parallel to $(\mathbf{i} - \mathbf{j})$, (c) the position vector of P at that time.

$a = \frac{v-u}{t} = \frac{(10\mathbf{i} + 10\mathbf{j}) - (-6\mathbf{i} - 14\mathbf{j})}{8} = (2\mathbf{i} + 3\mathbf{j})$
$v = (-6\mathbf{i} - 14\mathbf{j}) + t(2\mathbf{i} + 3\mathbf{j})$
Parallel to $(\mathbf{i} - \mathbf{j})$: $(2t - 6) \times -1 = (3t - 14) \times 1$, giving t = 4
$r = (2\mathbf{i} - \mathbf{j}) + 4(-6\mathbf{i} - 14\mathbf{j}) + \frac{1}{2} \times 4^2 (2\mathbf{i} + 3\mathbf{j}) = (-6\mathbf{i} - 33\mathbf{j})$
Answer: (a) $(2\mathbf{i} + 3\mathbf{j})$ m s^{-2} (b) t = 4 (c) $(-6\mathbf{i} - 33\mathbf{j})$ m

3 Equations in disguise (substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 12 = 0$.

Let $u = x^{\frac{1}{3}}$: $2u^2 + 2u - 12 = 0$.
$(u + 3)(u - 2) = 0$, so u = −3 or u = 2.
Cube: x = −27 or x = 8.
Answer: x = −27 or x = 8

4 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground with speed 24.5 m s^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$. Find (a) the time of flight, (b) the range, (c) the greatest height. Take g = 9.8 m s^{-2}.

Components: horizontal 19.6, vertical 14.7 (as $\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$)

Time of flight: $0 = 14.7t - 4.9t^2$, so $t = \frac{14.7}{4.9} = 3$

Range = 19.6 × 3 = 58.8

Greatest height: $0 = 14.7^2 - 2 \times 9.8 \times H$, so $H = \frac{216.09}{19.6} = 11.025$

Answer: (a) 3 s (b) 58.8 m (c) 11.025 m

5 Symmetric limits of a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Scores are N(64, 4^2). The highest 10% and the lowest 10% of scores are discarded.
Find the range of scores that remain, correct to 3 significant figures.

The remaining middle 80% lies between z = ±1.2816
Lower limit = 64 − 1.2816 × 2 = 61.44
Upper limit = 64 + 1.2816 × 2 = 66.56
Answer: 61.4 to 66.6

6 Implicit differentiation: tangent or normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x e^y + 3y = 3$.
Find the equation of the normal to C at the point (3, 0),
in the form $ax + by + c = 0$ with integers a, b, c.

Differentiate implicitly with respect to x

At (3, 0): $dy/dx = -\frac{1}{6}$

Normal gradient = $-1 \div (-\frac{1}{6}) = 6$

$y - 0 = 6(x - 3)$, then clear fractions

Answer: $6x - y - 18 = 0$

7 Binomial expansion (fractional or negative powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the binomial expansion of $(8 - x)^{\frac{2}{3}}$ in ascending powers of x up to and including the term in x^3.
State the values of x for which it is valid.

$(8 - x)^{\frac{2}{3}} = 8^{\frac{2}{3}}(1 - \frac{1}{8}x)^{\frac{2}{3}} = 4(1 + u)^{\frac{2}{3}}$, $u = -\frac{1}{8}x$.

$(1 + u)^{\frac{2}{3}} = 1 + \frac{2}{3}u + (-\frac{1}{9})u^2 + \frac{4}{81}u^3 + ...$

Substitute u and multiply by 4.
Valid when $|u| < 1$, i.e. $|x| < 8$.

Answer: $4 - \frac{1}{3}x - \frac{1}{144}x^2 - \frac{1}{2592}x^3 + ...$; $|x| < 8$

8 Exponential times trigonometric (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = e^{2x} \cos 2x$. Show that $dy/dx = e^{2x}(2 \cos 2x - 2 \sin 2x)$.
Find the x coordinate of the stationary point with the smallest positive x (3 s.f.), and its nature.

Product rule: $dy/dx = (2)e^{2x} \cos 2x - e^{2x} \times 2 \sin 2x$
$dy/dx = 0$ requires $2 \cos 2x = 2 \sin 2x$
$\tan 2x = 1$, smallest positive $2x = 0.7854$
$x = 0.393$
Second derivative test (or gradient change): a maximum
Answer: x = 0.393, maximum

9 Waiting for the r-th success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each shot Amira takes scores with probability $\frac{3}{4}$, independently.

(a) Find the probability that her fourth score comes on shot number 11.
(b) Find the probability that her fourth score comes on or before shot number 11.

(a) fourth score on shot 11: exactly 3 scores in the first 10 shots, then a score

$C(10, 3)(\frac{3}{4})^3(\frac{1}{4})^7 \times \frac{3}{4}$

= 0.00232

(b) On or before shot 11 means at least 4 scores in 11 shots: $X \sim B(11, \frac{3}{4})$

$P(X \geq 4) = 1 - P(X \leq 3) = 0.999$

Answer: (a) 0.00232 (b) 0.999

10 Lines in 3D: intersection and angle (CIE (9709); Further Maths on other boards)

Line l_1: $\mathbf{r} = \begin{pmatrix} 0 \\ -3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$,

line l_2: $\mathbf{r} = \begin{pmatrix} -4 \\ -4 \\ 2 \end{pmatrix} + t\begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix}$.

Show whether the lines meet, giving the point of intersection if they do, and find the acute angle between them to 1 d.p.

Equate components: $2s = -4 - 2t$; $-3 + s = -4$; $-1 - 3s = 2$

Solving the first two: $s = -1$, $t = -1$; the third equation is satisfied

Intersection: $(-2, -4, 2)$

$\cos\varphi = \frac{|-4|}{\sqrt{14}\sqrt{4}} = 0.5345$, $\varphi = 57.7°$

Answer: Lines meet at $(-2, -4, 2)$; angle 57.7°