Day 363 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Dropped and projected particles meet (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Ball A is projected vertically upwards from ground level with speed 37 m s^{-1}. At the same instant, ball B is released from rest 111 m directly above A.
The balls collide. Find (a) the height above the ground at which they collide, (b) the velocity of A just before the collision.
Take g = 9.8 m s^{-2}.

Relative to B, A moves upwards at the constant speed 37 m s^{-1} (both have acceleration g)
So the balls meet when 37 t = 111, giving t = 3
Height of A: $s = 37 \times 3 - \frac{1}{2} \times 9.8 \times 3^2 = 66.9$
$v = u - gt = 37 - 9.8 \times 3 = 7.6$
Answer: (a) 66.9 m (b) 7.6 m s^{-1} upwards

2 Simultaneous logarithmic equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
$\log_2 x - \log_2 y = 1$
$x + y = 9$

$\log_2(x/y) = 1$, so $x = 2^1 y = 2y$
$2y + y = 9$, so $3y = 9$
Answer: x = 6, y = 3

3 Shaded normal curve and the standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The pdf of $X \sim N(31, \sigma^2)$ is shown; the shaded area is symmetrical.
(a) Find σ. (b) Find P(X > 43).

The area 0.5 leaves 0.500 in total in the tails, 0.250 in each
$P(X < 22) = 0.250$ so $\frac{22 - 31}{\sigma} = -0.6745$
$\sigma = \frac{9}{0.6745} = 13.34$
P(X > 43) = P(Z > 0.899) = 0.184
Answer: (a) 13.3 (b) 0.184

4 Perpendicular lines and area (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the diagram, line l_1 passes through A and B.
Line l_2 is perpendicular to l_1 at B and meets the
x-axis at C. Find the coordinates of C and the area of triangle ABC.

Gradient of $l_1 = \frac{6 - 4}{-2 + 8} = \frac{1}{3}$.
Gradient of $l_2 = -3$. Equation: $y - 6 = -3(x + 2)$.
y = 0: x = 0, so C(0, 0).
AB = $2\sqrt{10}$, BC = $2\sqrt{10}$: area = ½ × AB × BC.
Answer: C(0, 0); area 20

5 Implicit differentiation: tangent or normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x e^y + 3y = 2$.
Find the equation of the normal to C at the point (2, 0),
in the form ax + by + c = 0 with integers a, b, c.

Differentiate implicitly with respect to x
At (2, 0): $dy/dx = -\frac{1}{5}$
Normal gradient = $-1 \div (-\frac{1}{5}) = 5$
$y - 0 = 5(x - 2)$, then clear fractions
Answer: 5x − y − 10 = 0

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6 Direction of motion at 45 degrees (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from level ground with horizontal and vertical velocity components 9.8 and 24.5 m s^{-1}.
Find (a) the two times at which it moves at 45° to the horizontal, (b) the horizontal distance between those two points.
Take g = 9.8 m s^{-2}.

At 45° to the horizontal, |v_y| = v_x = 9.8
Moving upwards: 24.5 − 9.8t = 9.8, so t = 1.5
Moving downwards: 24.5 − 9.8t = −9.8, so t = 3.5
Horizontal distance = 9.8 × (3.5 − 1.5)
Answer: (a) 1.5 s and 3.5 s (b) 19.6 m

7 Most likely value of a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(11, \frac{3}{10})$.

(a) Show that $P(X = r + 1) \div P(X = r) = \frac{11 - r}{r + 1} \times \frac{3}{7}$.

(b) Find the most likely value of X and its probability (3 s.f.).

$P(X = r + 1) \div P(X = r) = \frac{C(11, r + 1)}{C(11, r)} \times \frac{p}{q} = \frac{11 - r}{r + 1} \times \frac{p}{q}$

With $p = \frac{3}{10}$ and $q = \frac{7}{10}$, $p \div q = \frac{3}{7}$

The ratio exceeds 1 when $(11 - r)\frac{3}{10} > (r + 1)\frac{7}{10}$, that is r < 2.6

So probabilities rise up to r = 3 and then fall; the mode is 3
P(X = 3) = 0.257
Answer: 3 with probability 0.257

8 Rational inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{-3x + 4}{x + 3} \geq -2$.

Rearrange: $\frac{-3x + 4}{x + 3} - (-2) \geq 0$.

Combine: $\frac{-x + 10}{x + 3} \geq 0$.

Critical values: x = 10 (zero of numerator), x = −3 (excluded, denominator zero).
Sign analysis of the three regions gives the solution.
Answer: $-3 < x \leq 10$

9 Partial fractions then integrate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Express $\frac{5x + 9}{(x + 3)(x + 1)}$ in partial fractions.

(b) Hence find the exact value of $\int_0^1 \frac{5x + 9}{(x + 3)(x + 1)}\,dx$.

Let N/D = $\frac{A}{x + 3} + \frac{B}{x + 1}$: 5x + 9 ≡ A(x + 1) + B(x + 3)
x = −3 gives A = 3; x = −1 gives B = 2
Integral = [3 ln(x + 3) + 2 ln(x + 1)]
Substitute limits 0 and 1
Answer: (a) 3/(x + 3) + 2/(x + 1); (b) 8ln 2 − 3ln 3

10 Equations in disguise (substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x + 1\sqrt{x} - 2 = 0$.

Let $u = \sqrt{x}$: $u^2 + 1u - 2 = 0$.
(u − 1)(u + 2) = 0, so $\sqrt{x} = 1$ or $\sqrt{x} = -2$.
$\sqrt{x}$ cannot be negative, so reject $\sqrt{x} = -2$.
x = 1.
Answer: x = 1