Day 362 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Double angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3\cos 2\theta + 9\cos\theta + 6 = 0$
for $0 \leq \theta < 360°$.

Use $\cos 2\theta \equiv 2\cos^2\theta - 1$: $6\cos^2\theta + 9\cos\theta + 3 = 0$
$3(2\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = -1$

Using the CAST diagram for $0 \leq \theta < 360°$
Answer: θ = 120°, 180°, 240°

2 Coded regression line (AQA, Edexcel, OCR, MEI)

Data pairs (x, y) are coded by $u = x - 2$ and $v = \frac{y}{2}$. The regression line of v on u is $v = 12 + 0.4u$.

(a) Find the regression line of y on x in the form $y = a + bx$.
(b) Use it to estimate y when $x = 6$.

Substitute $u = x - 2$ and $v = y/2$: $\frac{y}{2} = 12 + 0.4(x - 2)$

$y = 2[12 + 0.4x - 0.8] = 2 \times 11.2 + 2 \times 0.4x$
$y = 22.4 + 0.8x$
At $x = 6$: $y = 22.4 + 0.8 \times 6 = 27.2$
Answer: (a) $y = 22.4 + 0.8x$ (b) 27.2

3 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A tennis ball is projected from a point O on horizontal ground with speed 49 m s^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Find (a) the greatest height, (b) the speed when it is at half of this height (3 s.f.). Take $g = 9.8$ m s^{-2}.

Vertical: $0 = 39.2^2 - 2 \times 9.8 \times H$, so $H = 78.4$

At height $\frac{H}{2} = 39.2$: $v_y^2 = 39.2^2 - 2 \times 9.8 \times 39.2 = 768.32$

$v_x = 29.4$ (constant), so speed^2 = 864.36 + 768.32 = 1632.68
Answer: (a) 78.4 m (b) 40.4 m s^{-1}

4 Arithmetic terms forming a geometric sequence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic sequence is 12 and its common difference d is non-zero. The 1st, 5th and 13th terms are the first three terms of a geometric sequence. Find d and the common ratio.

Terms: 12, 12 + 4d, 12 + 12d.
Geometric: $(12 + 4d)^2 = (12)(12 + 12d)$.
Expand: $16d^2 + 96d + 144 = 144d + 144$, so $d(16d - 48) = 0$.
$d \neq 0$, so $d = 3$. Ratio = $\frac{24}{12} = 2$.
Answer: d = 3, ratio = 2

5 Newton-Raphson iteration (AQA, Edexcel, OCR, MEI, Eduqas (Newton-Raphson))

$f(x) = x^3 + 3x - 11$.
(a) Show that $f(x) = 0$ has a root α with $1 < \alpha < 2$.
(b) Use Newton-Raphson with $x_0 = 1$ to find x_1 and x_2
to 4 decimal places.

$f(1) = -7$ and $f(2) = 3$: a change of sign, f continuous

$f'(x) = 3x^2 + 3$, so $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1 - \frac{-7.000}{6.000} = 2.1667$

$x_2 = 2.1667 - \frac{5.671}{17.08} = 1.8347$
Answer: x_1 = 2.1667, x_2 = 1.8347

6 Poisson approximation to a binomial (CIE)

X ~ B(200, 0.015).
(a) Explain why a Poisson approximation is suitable and state its distribution.
(b) Use it to estimate P(X > 3), to 4 s.f.
(c) Find the exact value of P(X > 3), to 4 s.f.

n = 200 is large and p = 0.015 is small, with np = 3
X is approximately Po(3)
$P(X > 3) \approx 1 - P(Y \leq 3) = 0.3528$
Exact: $1 - P(X \leq 3)$ with X ~ B(200, 0.015) = 0.3528
Answer: (a) Po(3) (b) 0.3528 (c) 0.3528

7 Stationary points of trigonometric curves (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact coordinates of the stationary points of
$y = x + 2\cos x$ for $0 \leq x \leq \pi$,
and determine their nature.

$dy/dx = 1 - 2\sin x = 0$, so $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

$d^2y/dx^2 = -2\cos x$
Its sign at each point decides maximum or minimum
Answer: $(\frac{\pi}{6}, \frac{1}{6}\pi + \sqrt{3})$ maximum; $(\frac{5\pi}{6}, \frac{5}{6}\pi - \sqrt{3})$ minimum

8 Circle through three points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through P(1, −1), Q(8, 0)
and R(4, 8). Find its equation.

Centre is on the perpendicular bisector of each chord.
Bisector of PQ: 7x + y = 31.
Bisector of QR: −4x + 8y = 8.
Solving: centre (4, 3).
$r^2 = (1 - 4)^2 + (-1 - 3)^2 = 25$.
Answer: $(x - 4)^2 + (y - 3)^2 = 25$

9 Constant speed up a slope, force at an angle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of weight 130 N is pulled at constant speed up a line of greatest slope of a rough plane ($\tan\alpha = \frac{5}{12}$, $\mu = \frac{7}{24}$) by a force P N at angle β above the plane.

(a) Find P when $\tan\beta = \frac{12}{5}$. (b) Find the least possible value of P as β varies.

Perpendicular to the plane: $R + P\sin\beta = 130\cos\alpha$, so $R = 130 \times \frac{12}{13} - P\sin\beta$
Along the plane: $P\cos\beta = 130\sin\alpha + \mu R$, so $P(\cos\beta + \mu\sin\beta) = 130(\sin\alpha + \mu\cos\alpha)$
(a) $P(\frac{5}{13} + \frac{7}{24} \times \frac{12}{13}) = 130(\frac{5}{13} + \frac{7}{24} \times \frac{12}{13})$, so P = 130

(b) P is least when $\cos\beta + \mu\sin\beta$ is greatest, i.e. $\sqrt{(1 + \mu^2)} = \frac{25}{24}$ (when $\tan\beta = \frac{7}{24}$)

$P_{min} = 130(\frac{5}{13} + \frac{7}{24} \times \frac{12}{13}) \div \frac{25}{24}$
Answer: (a) 130 N (b) 81.6 N

Day 362 **A Level Maths: S Tier 10-a-day: Model Solutions**

10 Arithmetic series from a sum formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The sum of the first n terms of a sequence is

$S_n = 4n^2 + 3n$.

(a) Find an expression for u_n.

(b) Find the least n for which $u_n > 178$.

$u_n = S_n - S_{n-1} = 4n^2 + 3n - [4(n-1)^2 + 3(n-1)]$

$= 8n - 1$.

$u_1 = S_1 = 7$ also fits, so $u_n = 8n - 1$ for all n.

$8n - 1 > 178 \Rightarrow n > \frac{179}{8}$, so n = 23.

Answer: (a) $u_n = 8n - 1$ (b) n = 23