Day 361 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of a apple is N(106, 4^2) g. A apple is 'heavy' if it exceeds 109.
(a) Find the probability that a apple is heavy.
(b) 12 apples are chosen at random. Find the probability that at least 4 are heavy.

(a) P(X > 109) = P(Z > 0.750) = 0.2266
(b) Let Y = number heavy, Y ~ B(12, 0.2266)
P(Y ≥ 4) = 1 − P(Y ≤ 3) = 0.280
Answer: (a) 0.227 (b) 0.280

2 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 450 kg, carrying a passenger of mass 60 kg, is supported by a vertical cable. The lift is moving upwards and accelerating at 0.4 m s^{-2}.
Find (a) the force the floor exerts on the passenger, (b) the tension in the cable. Take g = 9.8 m s^{-2}.

Take the direction of the acceleration as positive: it is upwards
Passenger: R − 60 × 9.8 = 60 × 0.4
R = 60 × (9.8 + 0.4) = 612
Lift and passenger: T − 510 × 9.8 = (450 + 60) × 0.4
T = 510 × 10.2 = 5202
Answer: (a) 612 N (b) 5202 N

3 Without replacement: find the number of counters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 3 green counters and some white counters, n in total.
Two are taken without replacement. P(at least one green) = $\frac{7}{12}$. Find n.

P(neither green) = $\frac{(n-3)}{n} \times \frac{(n-4)}{(n-1)} = \frac{5}{12}$

$\frac{5}{12}$ n(n − 1) = (n − 3)(n − 4)
Rearranging gives a quadratic in n whose roots are n = 9 and one other value
The other root is $\frac{16}{7}$ so is rejected (not a valid number of counters)
n = 9
Answer: n = 9

4 Cubic with only one real root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = $x^3 - x^2 + 5x + 7$.
Show that (x + 1) is a factor of f(x), and that the equation f(x) = 0 has exactly one real root.

f(−1) = −1 − 1 − 5 + 7 = 0, so (x + 1) is a factor.
f(x) = (x + 1)($x^2 - 2x + 7$).
For $x^2 - 2x + 7 = 0$: $b^2 - 4ac = 4 - 28 = -24 < 0$.
No real roots from the quadratic, so the only real root is x = −1.
Answer: Proved; the only real root is x = −1

5 Fixed point iteration and rearrangement (AQA, Edexcel, OCR, MEI, Eduqas, CIE (iterative methods))

The equation $x^3 = 2x + 14$ has a root α.
(a) Show that the equation can be written as x = $\sqrt[3]{(2x + 14)}$.
(b) Use $x_{n+1} = \sqrt[3]{(2x_n + 14)}$ with $x_0 = 2$ to find x_1, x_2, x_3
to 4 d.p., and hence state α to 3 d.p.

Take the cube root of both sides to obtain x = $\sqrt[3]{(2x + 14)}$
x_1 = 2.6207, x_2 = 2.6797, x_3 = 2.6851
The iteration converges (successive values agree to 3 d.p.): α = 2.686
Answer: x_1 = 2.6207, x_2 = 2.6797, x_3 = 2.6851; α = 2.686

6 Velocity with a square-root term (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a straight line. At time t seconds, where $t \geq 0$, its velocity is v m s^{-1}, where $v = 10\sqrt{t} - 2t$.
Find (a) the greatest speed of P, (b) the greatest displacement of P from its starting point.

$\frac{dv}{dt} = \frac{10}{2\sqrt{t}} - 2 = 0$ when $\sqrt{t} = \frac{5}{2}$, so $t = \frac{25}{4}$

$v = 10 \times \frac{5}{2} - 2 \times \frac{25}{4} = \frac{25}{2}$

$v = 0$ when $\sqrt{t}(10 - 2\sqrt{t}) = 0$: $t = 25$ ($t = 0$ is the start)

$s = \int v\,dt = \frac{2}{3} \times 10\, t^{\frac{3}{2}} - \frac{2}{2}t^2$, so at $t = 25$: $s = \frac{2500}{3} - 625$

Answer: (a) $\frac{25}{2}$ m s^{-1} (b) $\frac{625}{3}$ m

7 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 - xy + y^2 = 27$.
Find the coordinates of the points on C where the tangent is parallel to the x-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + -y}{-x + 2y}$

Horizontal tangent when $2x + -y = 0$, so $y = \frac{2x}{1}$

Substitute into the equation of C: $x^2 + (-xy) + (y^2) = 27$
Solve for the remaining coordinate
Answer: $(-3, -6)$ and $(3, 6)$

8 Geometric series: least n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 2 and common ratio 5.
Find the least n for which $S_n > 46347$.

$S_n = \frac{2(5^n - 1)}{4} > 46347$.

$5^n > 92695$.

$n > \frac{\log(9.27e+04)}{\log(5)} = 7.106$.

So the least n is 8.
Answer: n = 8

9 Triangle in 3D: angle and area (CIE (9709); Further Maths on other boards)

The points A, B, C have coordinates $(-1, 4, 2)$, $(-3, -1, -2)$, $(1, 1, -1)$.
Find angle ABC to 1 d.p., and the exact area of triangle ABC.

$BA = \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}$, $BC = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$; $BA \cdot BC = 22$

$|BA| = \sqrt{45}$, $|BC| = \sqrt{21}$

$\cos ABC = \frac{22}{\sqrt{45}\sqrt{21}} = 0.7157$, so $ABC = 44.3°$

$|BA \times BC| = \left|\begin{pmatrix} -3 \\ 14 \\ -16 \end{pmatrix}\right| = \sqrt{461}$

$\text{Area} = \frac{1}{2}|BA \times BC|$

Answer: ABC = 44.3°, area = $\frac{1}{2}\sqrt{461}$

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10 Remainder theorem (quadratic divisor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

When a polynomial f(x) is divided by (x + 3) the remainder is 1; when divided by (x + 1) the remainder is −1. Find the remainder when f(x) is divided by (x + 3)(x + 1).

f(x) = (x + 3)(x + 1)Q(x) + px + q.
f(−3) = 1: −3p + q = 1.
f(−1) = −1: −p + q = −1.
Subtracting: −2p = 2, so p = −1, q = −2.
Answer: −x − 2