Day 360 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Indices: simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve simultaneously, giving integer x and y:

$16^x \times 2^y = 512$

$27^x \div 9^y = 81$

Write each side with prime bases: $512 = 2^9$, $81 = 3^4$.

First: $2^{4x} \times 2^{1y} = 2^9$, so $4x + 1y = 9$.

Second: $3^{3x - 2y} = 3^4$, so $3x - 2y = 4$.

Solving the pair of linear equations: $x = 2$, $y = 1$.

Answer: $x = 2$, $y = 1$

2 Compound change with logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A colony of 1200 cells grows by 10% each day.

Find the least whole number of days for the colony to exceed 3600 cells.

$1200 \times 1.10^n > 3600$, so $1.10^n > 3$

$n > \dfrac{\ln 3}{\ln 1.10} = 11.53$

n must be a whole number, so $n = 12$

Answer: 12

3 Grouped median with an unknown frequency (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the times, t minutes, taken by some runners.

Using linear interpolation, the median is estimated as 25 minutes. Find the value of k.

Total = 22 + k, so the median class is 20 to 30

$$\text{Median} = 20 + \frac{\frac{(22 + k)}{2} - (3 + k)}{7} \times 10 = 25$$

Simplify: $8 - k/2 = 3.5$

$k = 9$

Answer: $k = 9$

4 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ax^n$ passes through (9, 24) and (16, 32).

Find the values of a and n.

$24 = a \times 9^n$ and $32 = a \times 16^n$

Divide: $\left(\frac{16}{9}\right)^n = \frac{4}{3}$ so $n = \frac{1}{2}$

$a = 24 \div 9^{\frac{1}{2}} = 8$

Answer: $a = 8$, $n = \frac{1}{2}$

5 Circle through three points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(9, 0), B(−4, 13)

and C(−9, −12). Find its equation.

Centre is on the perpendicular bisector of each chord.

Bisector of AB: $-13x + 13y = 52$.

Bisector of BC: $-5x - 25y = 20$.

Solving: centre (−4, 0).

$r^2 = (9 - (-4))^2 + (0 - 0)^2 = 169$.

Answer: $(x + 4)^2 + (y + 0)^2 = 169$

6 Car, trailer and tow bar (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 900 kg tows a trailer of mass 400 kg along a horizontal road, with driving force 1030 N. Resistances are 400 N on the car and 240 N on the trailer.

(a) Find the tension in the tow bar. At 3.6 m s^{-1} the tow bar breaks. (b) Find the distance between car and trailer when the trailer stops.

Whole system: $a = \frac{1030 - 400 - 240}{1300} = 0.3$

Trailer: $T - 240 = 400 \times 0.3$, so $T = 360$

After the break the trailer decelerates at $\frac{240}{400} = 0.6$, and stops after $t = \frac{3.6}{0.6} = 6$ s

Trailer travels $\frac{3.6}{2} \times 6 = 10.8$ m

Car: $a = \frac{1030 - 400}{900} = 0.7$, so $s = 3.6 \times 6 + ½ \times 0.7 \times 6^2 = 34.2$ m

Distance apart = 34.2 − 10.8

Answer: (a) 360 N (b) 23.4 m

7 Conditional probability from given values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, P(A) = 0.5, P(B) = 0.55 and P(A | B) = 0.25.

(a) Find P(A ∩ B).

(b) Find P(A′ | B′).

$P(A \cap B) = P(A \mid B) \times P(B) = 0.25 \times 0.55 = 0.1375$

$P(A \cup B) = 0.5 + 0.55 - 0.1375 = 0.9125$

$P(B') = 0.45$

$P(A' \cap B') = 1 - P(A \cup B) = 0.0875$

$P(A' \mid B') = \frac{0.0875}{0.45}$

Answer: (a) 0.1375 (b) $\frac{7}{36}$

8 Particle on a rough table, string slack when B lands (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (5 kg) and B (3 kg) are joined by a light string over a smooth pulley (see diagram). A lies on a rough table with μ = 0.4, and B is 0.8 m above the floor.

The system is released from rest. Find the total distance A moves before coming to rest (A does not reach the pulley).

Until B lands, A: $T - 0.4 \times 5 \times 9.8 = 5a$. B: $3 \times 9.8 - T = 3a$

Adding: $(3 - 0.4 \times 5) \times 9.8 = 8a$, so $a = 1.225$

$v^2 = 2 \times 1.225 \times 0.8 = 1.96$, so $v = 1.4$

After B lands, A decelerates at $\mu g = 3.92$: extra distance $= \frac{1.4^2}{2 \times 3.92} = 0.25$

Total distance = 0.8 + 0.25

Answer: 1.05 m

9 Binomial expansion (integer powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of

$(2x - 1)(1 - 3x)^7$.

$(1 - 3x)^7$: x^3 term has coefficient $C(7,3) \times (-3)^3 = -945$, x^2 term has coefficient $C(7,2) \times (-3)^2 = 189$.

x^3 in the product: $(-1) \times (-945) + 2 \times 189$.

= 1323.

Answer: 1323

10 Volumes of revolution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region bounded by $y = 3\sqrt{\sin x}$, the x-axis and the lines $x = 0$ and $x = \pi$ is rotated through 2π about the x-axis.
Find the exact volume of the solid formed.

$V = \pi\int_0^{\pi} y^2\,dx = \pi\int_0^{\pi} (3\sqrt{\sin x})^2\,dx$

Simplify the integrand, then integrate

$V = \pi \times (18)$

Answer: 18π