Day 359 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Shaded normal curve and the standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The pdf of $X \sim N(89, \sigma^2)$ is shown; the shaded area is symmetrical.
(a) Find σ. (b) Find $P(X > 112)$.

The area 0.8 leaves 0.200 in total in the tails, 0.100 in each

$P(X < 74) = 0.100$ so $\frac{74 - 89}{\sigma} = -1.2816$

$\sigma = \frac{15}{1.2816} = 11.70$

$P(X > 112) = P(Z > 1.965) = 0.0247$

Answer: (a) 11.7 (b) 0.0247

2 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows y = f(x), made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of
$y = 3f(-2x + 4)$.

Inside f: replace x by $-2x + 4$, so x-coordinates become $(x - 4) \div (-2)$.
Outside: y-coordinates become $3 \times y$.
A: (3, 6) B: (2, 9)
C: (0, 6) D: $(-1, -9)$
Answer: A′(3, 6), B′(2, 9), C′(0, 6), D′(−1, −9)

3 Stone thrown from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 8 m s^{-1} from the top of a cliff. The top of the cliff is 46.4 m above the sea. Find (a) the time taken for the stone to reach the sea, (b) the total distance travelled by the stone, to 3 s.f. Take g = 9.8 m s^{-2}.

Take upwards as positive: $-46.4 = 8t - 4.9t^2$, so $4.9t^2 - 8t - 46.4 = 0$

$t = \frac{8 + \sqrt{(8^2 + 19.6 \times 46.4)}}{9.8} = \frac{8 + 31.2}{9.8} = 4$

Height risen above the cliff top $= \frac{8^2}{(2 \times 9.8)} = 3.27$ m

Total distance $= 2 \times 3.27 + 46.4$

Answer: (a) 4 s (b) 52.9 m

4 Equations quadratic in an exponential (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $5^{2x} - 6 \times 5^{x+1} + 125 = 0$,
giving each answer exactly and, where not an integer, to 3 s.f.

$5^{x+1} = 5 \times 5^x$. Let $u = 5^x$: $u^2 - 30u + 125 = 0$

$(u - 5)(u - 25) = 0$, so $5^x = 5$ or $5^x = 25$

Take logarithms: $x = \log_5 5 = 1$; $x = \log_5 25 = 2$

Answer: $x = \log_5 5 = 1$; $x = \log_5 25 = 2$

5 Equations with a square root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sqrt{x + 9} = x + 7$.

Square both sides: $x + 9 = (x + 7)^2$.

$x^2 + 13x + 40 = 0$, so $x = -8$ or $x = -5$.

Check in $\sqrt{x + 9} = x + 7$: $x = -5$ gives $2 \geq 0$ (valid).
$x = -8$ gives right-hand side $-1 < 0$, so it is rejected.

Answer: $x = -5$

6 Force up a rough slope: find weight and friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of weight W N is on a rough plane inclined at α to the horizontal, where $\sin\alpha = \frac{12}{13}$. A force P N acts up a line of greatest slope; the particle stays at rest for $78 \le P \le 114$.
Find (a) W, (b) the coefficient of friction, (c) the acceleration when P = 228. Take g = 9.8 m s^{-2}.

Limiting friction acts down the plane at P = 114 and up the plane at P = 78
$P_{max} = W\sin\alpha + F$ and $P_{min} = W\sin\alpha - F$, where $F = \mu W\cos\alpha$

Adding: $2W\sin\alpha = 192$, so $W = \frac{192}{2 \times \frac{12}{13}} = 104$

Subtracting: $2F = 36$, so $F = 18 = \mu \times 104 \times \frac{5}{13}$, giving $\mu = 0.45$

P = 228: net force up the plane = 228 − 114 = 114, and mass = $\frac{104}{9.8}$

Answer: (a) 104 N (b) 0.45 (c) 10.7 m s^{-2}

7 Volumes of revolution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region bounded by $y = \sin x + \cos x$, the x-axis and the lines
$x = 0$ and $x = \pi/2$ is rotated through 2π about the x-axis.
Find the exact volume of the solid formed.

$V = \pi\int_0^{\pi/2} y^2\,dx = \pi\int_0^{\pi/2} (\sin x + \cos x)^2\,dx$

Simplify the integrand, then integrate

$V = \pi \times (1 + \frac{1}{2}\pi)$

Answer: $\frac{1}{2}\pi(2 + \pi)$

8 Perpendicular lines and area (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the diagram, line l_1 passes through A and B.
Line l_2 is perpendicular to l_1 at B and meets the
x-axis at C. Find the coordinates of C and the area of triangle ABC.

Gradient of $l_1 = \frac{9-8}{3-6} = -\frac{1}{3}$.
Gradient of $l_2 = 3$. Equation: $y - 9 = 3(x - 3)$.
$y = 0$: $x = 0$, so C(0, 0).
AB = $\sqrt{10}$, BC = $3\sqrt{10}$: area = ½ × AB × BC.
Answer: C(0, 0); area 15

9 Exponential and power models by logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data satisfy $y = ab^x$. The line of best fit for lg y against x is $\lg y = 0.8 + 0.85x$.
(a) Find a and b to 3 significant figures.
(b) Estimate y when x = 15, to 3 significant figures.
(c) Interpret the value of b.

$y = ab^x$ gives $\lg y = \lg a + x \lg b$
$\lg a = 0.8$ so $a = 10^{0.8} = 6.310$
$\lg b = 0.85$ so $b = 10^{0.85} = 7.079$
$x = 15$: $\lg y = 0.8 + 0.85 \times 15 = 13.55$
$y = 10^{13.55} = 35500000000000$
b is the factor y is multiplied by each time x increases by 1 (growth of 610%)
Answer: (a) a = 6.31, b = 7.08 (b) 35500000000000 (c) y grows by a factor of 7.08 per unit of x

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10 Lines in 3D: intersection and angle (CIE (9709); Further Maths on other boards)

Line l_1: $\mathbf{r} = \begin{pmatrix} 0 \\ -5 \\ 7 \end{pmatrix} + s\begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$,

line l_2: $\mathbf{r} = \begin{pmatrix} -5 \\ -2 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ -1 \\ -3 \end{pmatrix}$.

Show whether the lines meet, giving the point of intersection if they do, and find the acute angle between them to 1 d.p.

Equate components: $-s = -5 - 3t$; $-5 + 2s = -2 - t$; $7 - 2s = 2 - 3t$

First two give $s = 2$, $t = -1$, but these fail in the third equation, so the lines do not meet (skew)

$\cos\varphi = \frac{|7|}{\sqrt{9}\sqrt{19}} = 0.5353$, $\varphi = 57.6°$

Answer: Lines do not meet (skew); angle 57.6°