Day 358 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Conditional probability from given values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, P(A) = 0.5, P(B) = 0.65 and P(A | B) = 0.6.
(a) Find P(A ∩ B).
(b) Find P(A′ | B′).

P(A ∩ B) = P(A | B) × P(B) = 0.6 × 0.65 = 0.39
P(A ∪ B) = 0.5 + 0.65 − 0.39 = 0.76
P(B′) = 0.35
P(A′ ∩ B′) = 1 − P(A ∪ B) = 0.24
$P(A' \mid B') = \frac{0.24}{0.35}$

Answer: (a) 0.39 (b) $\frac{24}{35}$

2 Resultant of forces: unknown constant and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-6\mathbf{i} - 5\mathbf{j})$ N, $(6\mathbf{i} + k\mathbf{j})$ N and $(-4\mathbf{i} - 5\mathbf{j})$ N act on a particle of mass 5 kg. The resultant is parallel to $(-4\mathbf{i} - 9\mathbf{j})$.
Find (a) the value of k, (b) the magnitude of the acceleration of the particle, to 3 s.f.

Resultant = $(-4)\mathbf{i} + (-10 + k)\mathbf{j}$
Parallel to $(-4\mathbf{i} - 9\mathbf{j})$: $\frac{-10 + k}{-4} = \frac{-9}{-4}$, so k = 1
R = $(-4\mathbf{i} - 9\mathbf{j})$ N, so $|R| = \sqrt{(-4^2 + -9^2)} = 9.849$
a = |R| ÷ 5
Answer: (a) k = 1 (b) 1.97 m s^{-2}

3 Normal test: critical region and Type II error (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ N(μ, 25). H_0: μ = 69 is tested against H_1: μ > 69 using the mean of a sample of size 36 at the 1% level.
(a) Find the critical region for the sample mean.
(b) Find the probability of a Type II error when μ = 70.

Under H_0, sample mean ~ $N(69, \frac{25}{36})$, standard error = 0.8333
z = 2.3263 for the upper tail: critical value = 69 + 2.3263 × 0.8333 = 70.94
Type II error: do not reject H_0, sample mean < 70.94, where mean ~ N(70, 0.8333)
$z = \frac{70.94 - 70}{0.8333} = 1.126$
Probability = 0.870
Answer: (a) sample mean > 70.9 (b) 0.870

4 Simultaneous logarithmic equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
$\log_2 x - \log_2 y = 1$
$x + y = 21$

$\log_2(x/y) = 1$, so $x = 2^1 y = 2y$
2y + y = 21, so 3y = 21
Answer: x = 14, y = 7

5 Simultaneous equations (linear and quadratic) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve simultaneously:
$x + y = -3$
$3x^2 - 2xy - y^2 = 7$

From the first equation x = −3 − y.
Substitute into the second equation and collect terms:
this gives $4y^2 + 24y + 20 = 0$.
y = −5 or y = −1, giving x = 2 or x = −2.
Answer: (x, y) = (2, −5) and (−2, −1)

Day 358 **A Level Maths: S Tier 10-a-day: Model Solutions**

6 Implicit differentiation: tangent or normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2y + xy^2 = -6$.
Find the equation of the normal to C at the point $(3, -1)$,
in the form $ax + by + c = 0$ with integers a, b, c.

Differentiate implicitly with respect to x

At $(3, -1)$: $dy/dx = \frac{5}{3}$

Normal gradient $= -1 \div \frac{5}{3} = -\frac{3}{5}$

$y - -1 = (-\frac{3}{5})(x - 3)$, then clear fractions

Answer: $3x + 5y - 4 = 0$

7 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^3 + 3x^2 - 4 < 0$.

Use the factor theorem to find roots: -2, 1.
$f(x) = (x + 2)^2(x - 1)$.
Sign of f in each region (left to right): $-$, $-$, $+$.
Take the regions where $f < 0$ (include roots if $\leq$ or $\geq$).
Answer: $x < -2$ or $-2 < x < 1$

8 Distance travelled with a change of direction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time t seconds its velocity is v m s^{-1}, where
$v = 3t^2 - 15t + 12$ for $0 \leq t \leq 6$
Find (a) the times at which P is at instantaneous rest, (b) the total distance travelled by P in the first 6 seconds.

$v = 3(t - 1)(t - 4)$, so $v = 0$ when $t = 1$ and $t = 4$
$v > 0$ for $0 < t < 1$, $v < 0$ for $1 < t < 4$, $v > 0$ for $t > 4$: direction changes at each

$s = \int v\,dt = \frac{3}{3}t^3 - 7.5t^2 + 12t$

Distances: 0 to 1: $\frac{11}{2}$; 1 to 4: $\frac{27}{2}$; 4 to 6: 26

Total $= \frac{11}{2} + \frac{27}{2} + 26$

Answer: (a) $t = 1$, $t = 4$ (b) 45 m

9 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 + xy + 2y^2 = 63$.
Find the coordinates of the points on C where the
tangent is parallel to the x-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + y}{x + 4y}$

Horizontal tangent when $2x + y = 0$, so $y = -\frac{2x}{1}$

Substitute into the equation of C: $x^2 + (xy) + (2y^2) = 63$
Solve for the remaining coordinate
Answer: $(3, -6)$ and $(-3, 6)$

10 Line tangent to a hyperbola (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = \frac{-2x + 10}{x - 1}$.

(a) Write down the equations of the asymptotes of C.
(b) The line $y = -2x + k$ is a tangent to C.
Find the possible values of k.

(a) Vertical: $x = 1$. As $x \to \infty$, $y \to -2$, so $y = -2$.
(b) $-2x + 10 = (-2x + k)(x - 1)$.
$-2x^2 + (k + 4)x + (-k - 10) = 0$.
Tangent: $b^2 - 4ac = 0$, which simplifies to $k^2 - 64 = 0$.
Answer: (a) $x = 1$, $y = -2$ (b) $k = -8$ or $k = 8$