Day 357	A Level Maths: S Tier 10-a-day
1 Two drones P and Q move so that, t seconds after launch, their position vectors in metres are $\mathbf{r}_P = \begin{pmatrix} -3 \\ 0 \\ -3 \end{pmatrix} + t\begin{pmatrix} -3 \\ -1 \\ -2 \end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix} -1 \\ -3 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix}$. Find the time at which they are closest and the exact minimum distance between them.	
2 Each shot Amira takes scores with probability $\frac{3}{5}$, independently. (a) Find the probability that her fourth score comes on shot number 9. (b) Find the probability that her fourth score comes on or before shot number 9.	
3 Particles A and B are projected from a point O at the same instant, with initial velocities $(8\mathbf{i} + 47\mathbf{j})$ m s^{-1} and $(3\mathbf{i} + 35\mathbf{j})$ m s^{-1} respectively (i horizontal, j vertically upwards). Both are still in flight after 4 s. Find (a) the distance AB at this time, (b) the angle AB makes with the horizontal, to 1 d.p.	
4 Use the trapezium rule with 5 strips to estimate $\int_1^2 \sqrt{1 + x^3}\, dx$, to 4 s.f. Is it an overestimate or an underestimate? Explain.	
5 The line $6y = 8x + k$ is a tangent to the circle $(x + 4)^2 + (y + 1)^2 = 4$. Find the two possible values of k.	

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6 A population of bacteria is modelled by $P = P_0e^{kt}$, t in hours. Initially P = 500 and after 3 hours P = 1000. Find k, and the time for the population to reach 1000, to 3 s.f.	
7 $f(x) = \frac{12}{x^2 - 4x + 9}$ for all real x. Find the greatest value of f(x), the value of x at which it occurs, and the range of f.	
8 A geometric series has first term 4 and common ratio $\frac{3}{2}$. Find the least n for which $S_n > 77212$.	
9 A test of H_0: p = 0.5 against H_1: p < 0.5 uses a sample of 20 at the 10% level. (a) Find the critical region for X, the number of successes. (b) Find the actual significance level. (c) Find P(Type II error) when in fact p = 0.3.	
10 Ball A is projected vertically upwards from ground level with speed 39 m s^{-1}. At the same instant, ball B is released from rest 39 m directly above A. The balls collide. Find (a) the height above the ground at which they collide, (b) the velocity of A just before the collision. Take g = 9.8 m s^{-2}.	