Day 355 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 15 and upper quartile 22. An outlier is a value more than 1.5 × IQR beyond the nearest quartile.

(a) Find the range of values that are not outliers.

(b) 8 observations have mean 20. The largest is 37, an outlier, and it is removed. Find the new mean.

IQR = 22 − 15 = 7, so 1.5 × IQR = 10.5

Lower limit = 15 − 10.5 = 4.5, upper limit = 22 + 10.5 = 32.5

$\Sigma x = 8 \times 20 = 160$, so without 37: $\Sigma x = 123$

New mean = $\frac{123}{7}$

Answer: (a) 4.5 to 32.5 (b) 17.57

2 Three-stage journey (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train accelerates uniformly from rest at 2 m s^{-2} to a speed of V m s^{-1}, travels at this speed, then decelerates uniformly at 2.8 m s^{-2} to rest.

The journey takes 58 s and covers 728 m. Find V and the time spent at constant speed.

Time accelerating = $\frac{V}{2}$, time decelerating = $\frac{V}{2.8}$, so time at constant speed = $58 - \frac{V}{2} - \frac{V}{2.8}$

Area under the speed-time graph: $728 = 58V - \frac{V^2}{4} - \frac{V^2}{5.6}$

Multiply by 2ab: $4.8V^2 - 649.6V + 8153.6 = 0$

V = 14 or V = $\frac{364}{3}$

V = $\frac{364}{3}$ gives time at constant speed −46 s, which is negative, so V = 14

Time at constant speed = 58 − 7 − 5 = 46 s

Answer: V = 14, 46 s

3 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation

$y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes x = 2 and y = 2, so p = −2 and q = 2.

$y = \frac{a}{x - 2} + 2$.

P(3, −2): $-2 = \frac{a}{1} + 2$, so a = −4.

Answer: a = −4, p = −2, q = 2

4 Segments: exact area and perimeter (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A chord AB of a circle, centre O and radius 9 cm, subtends an angle of $\frac{\pi}{3}$ at O.

Find the exact area and the exact perimeter of the minor segment cut off by AB.

Sector OAB: $\frac{1}{2}r^2\theta = \frac{1}{2} \times 9^2 \times \frac{\pi}{3}$; triangle OAB: $\frac{1}{2}r^2\sin\theta = \frac{81}{4}\sqrt{3}$

Area = $\frac{27}{2}\pi - \frac{81}{4}\sqrt{3}$

Chord AB = $2r\sin(\theta/2) = 9$

Arc AB = $r\theta = 3\pi$

Perimeter = arc + chord

Answer: Area = $\frac{27}{2}\pi - \frac{81}{4}\sqrt{3}$ cm^2, perimeter = $9 + 3\pi$ cm

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5 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations
$x = -3 + 5\cos t$, $y = -2 + 5\sin t$.
Find its Cartesian equation.

$\cos t = \frac{x+3}{5}$, $\sin t = \frac{y+2}{5}$.

Use $\cos^2 t + \sin^2 t = 1$:

$(x+3)^2 + (y+2)^2 = 25$.

Answer: $(x+3)^2 + (y+2)^2 = 25$

6 Triple angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE (triple angle))

Given that $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$, solve
$\cos 3\theta = \cos\theta$ for $0 < \theta < 360°$.

Substitute: $\cos\theta(4\cos^2\theta - 3 - 1) = 0$

$\cos\theta = 0$ or $\cos^2\theta = 1$

$\cos\theta = 0$ gives 90°, 270°

$\cos^2\theta = 1$ gives 180°

Answer: θ = 90°, 180°, 270°

7 Large data set: linear change of units (AQA, Edexcel, OCR, MEI, Eduqas)

Large Data Set, daily mean air temperature (°C), Heathrow, September 2015 (n = 30): mean 15.3, sd 2.5. ($F = 1.8C + 32$)
(a) Find the mean and sd in °F.
(b) Is a reading of 22 more than 2 sd above the mean?

New value = 1.8 × old + 32, so mean = 1.8 × 15.3 + 32 = 59.54

The constant shifts the mean but not the spread; only the multiplier affects σ

$\sigma = 1.8 \times 2.5 = 4.500$

Outlier cut-off = mean + 2σ = 15.3 + 5 = 20.3 (original units)

22 is greater than 20.3, so yes

Answer: (a) mean 59.5, sd 4.50 (b) yes

8 Geometric series: least n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 6 and common ratio 5.
Find the least n for which $S_n > 83364$.

$S_n = \frac{6(5^n - 1)}{4} > 83364$.

$5^n > 55577$.

$n > \frac{\log(5.558e+04)}{\log(5)} = 6.788$.

So the least n is 7.

Answer: n = 7

9 Triangle in 3D: angle and area (CIE (9709); Further Maths on other boards)

The points A, B, C have coordinates (2, 1, −2), (4, 5, 2), (1, 1, 3).

Find angle ABC to 1 d.p., and the exact area of triangle ABC.

$\text{BA} = \begin{pmatrix} -2 \\ -4 \\ -4 \end{pmatrix}$, $\text{BC} = \begin{pmatrix} -3 \\ -4 \\ 1 \end{pmatrix}$; $\text{BA} \cdot \text{BC} = 18$

$|\text{BA}| = \sqrt{36}$, $|\text{BC}| = \sqrt{26}$

$\cos \text{ABC} = \frac{18}{\sqrt{36}\sqrt{26}} = 0.5883$, so $\text{ABC} = 54.0°$

$|\text{BA} \times \text{BC}| = \left|\begin{pmatrix} -20 \\ 14 \\ -4 \end{pmatrix}\right| = \sqrt{612}$

$\text{Area} = \frac{1}{2}|\text{BA} \times \text{BC}|$

Answer: ABC = 54.0°, area = $3\sqrt{17}$

10 Hinged rod supported by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (4 m, 80 N) is hinged at A to a vertical wall and held horizontal by a string BC, C on the wall above A, with $\tan \alpha = \frac{5}{12}$, where α is the angle between string and rod. A 30 N particle hangs at P, AP = 2 m.

Find (a) the tension, (b) the magnitude and direction of the hinge force (3 s.f. and 1 d.p.).

$\sin \alpha = \frac{5}{13}$, $\cos \alpha = \frac{12}{13}$. Moments about A: $T \sin \alpha \times 4 = 80 \times 2 + 30 \times 2$

$T \times \frac{5}{13} \times 4 = 220$, so $T = 143$

Horizontal: $H = T \cos \alpha = 132$. Vertical: $V + T \sin \alpha = 80 + 30$, so $V = 55$

Force $= \sqrt{(132^2 + 55^2)} = 143$, angle $= \tan^{-1}(\frac{55}{132}) = 22.6°$ above the horizontal

Answer: (a) 143 N (b) 143 N at 22.6° above the horizontal (away from the wall)