Day 354 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows y = f(x), made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of
y = 3f(−x + 1).

Inside f: replace x by −x + 1, so x-coordinates become (x − 1) ÷ (−1).
Outside: y-coordinates become 3 × y.
A: (7, 12) B: (5, 3)
C: (3, 9) D: (−3, 3)
Answer: A′(7, 12), B′(5, 3), C′(3, 9), D′(−3, 3)

2 Double angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cos 2\theta - \sin\theta = 0$
for $0 \leq \theta < 2\pi$, giving answers in terms of π.

Use $\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta + \sin\theta - 1 = 0$
$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$

Principal values and symmetry in $[0, 2\pi)$

Answer: $\boldsymbol{\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

3 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations

$x = 3t - 4$, $y = \frac{1}{t} + 2$, $t \neq 0$.

Find a Cartesian equation of the curve.

$t = \frac{x + 4}{3}$.

$y - 2 = \frac{1}{t} = \frac{1 \times 3}{x + 4}$.

Rearrange to remove the fraction.
Answer: (x + 4)(y − 2) = 3

4 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = t^2$, $y = 2t^3 + 12t$.
Find the coordinates of the points on the curve where the gradient is 9.

$dx/dt = 2t$, $dy/dt = 6t^2 + 12$, so $dy/dx = \frac{6t^2 + 12}{2t}$

Set equal to 9: $6t^2 - 18t + 12 = 0$
$t = 1$ or $t = 2$
Answer: (1, 14) and (4, 40)

5 Car, trailer and tow bar (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 1200 kg tows a trailer of mass 400 kg along a horizontal road, with driving force 2000 N. Resistances are 800 N on the car and 400 N on the trailer.

(a) Find the tension in the tow bar. At 12 m s^{-1} the tow bar breaks. (b) Find the distance between car and trailer when the trailer stops.

Whole system: $a = \frac{2000 - 800 - 400}{1600} = 0.5$

Trailer: $T - 400 = 400 \times 0.5$, so $T = 600$

After the break the trailer decelerates at $\frac{400}{400} = 1$, and stops after $t = \frac{12}{1} = 12$ s

Trailer travels $\frac{12}{2} \times 12 = 72$ m

Car: $a = \frac{2000 - 800}{1200} = 1$, so $s = 12 \times 12 + \frac{1}{2} \times 1 \times 12^2 = 216$ m

Distance apart = $216 - 72$

Answer: (a) 600 N (b) 144 m

6 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A coin is said to score a hit with probability 0.4. In 25 trials it does so 3 times.

Test at the 10% level whether the probability is different from 0.4.

H_0: $p = 0.4$, H_1: $p \neq 0.4$ (two-tailed). $X \sim B(25, 0.4)$

3 is below the mean 10, so use the lower tail: $P(X \leq 3) = 0.00237$

Each tail is compared with 5%: 0.00237 is less than 5%

Reject H_0: there is sufficient evidence that the probability is different from 0.4

Answer: Reject H_0 (p-value 0.00237 against 5%)

7 Displacement of the form R sin(ωt + φ) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a line. Its displacement from a fixed point O at time t seconds is s metres, where $s = 5 \sin 3t + 12 \cos 3t$.

Find (a) the greatest distance of P from O, (b) the greatest speed of P, (c) the first time P is at its greatest distance from O (3 s.f.).

Write $s = R \sin(3t + \varphi)$ with $R \cos \varphi = 5$ and $R \sin \varphi = 12$

$R = \sqrt{(5^2 + 12^2)} = 13$ and $\tan \varphi = \frac{12}{5}$, so $\varphi = 1.176$ rad

Greatest distance = R = 13

$v = \frac{ds}{dt} = 13 \times 3 \cos(3t + \varphi)$, so greatest speed = 39

s is greatest when $3t + \varphi = \frac{\pi}{2}$: $t = (1.5708 - 1.176) \div 3$

Answer: (a) 13 m (b) 39 m s^{-1} (c) 0.132 s

8 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-4x + 1| < |-2x + 5|$.

Square both sides: $(-4x + 1)^2 < (-2x + 5)^2$.

$(-2x - 4)(-6x + 6) < 0$.

Critical values $x = -2$ and $x = 1$.

The quadratic has positive coefficient of x^2, so choose the inner region.

Answer: $-2 < x < 1$

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9 Points of inflection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coordinates of the points of inflection of

$y = \frac{1}{x^2 + 3}$.

$dy/dx = -\frac{2x}{(x^2 + 3)^2}$

$d^2y/dx^2 = \frac{2(3x^2 - 3)}{(x^2 + 3)^3}$

Zero when $x^2 = 1$, so $x = \pm 1$, and the sign changes either side

$y = \frac{1}{4} = \frac{1}{4}$

Answer: $(1, \frac{1}{4})$ and $(-1, \frac{1}{4})$

10 Cumulative distribution function (CIE)

The cdf of X is $F(x) = 0$ for $x < 0$, $F(x) = \frac{x^4}{256}$ for $0 \leq x \leq 4$, and $F(x) = 1$ for $x > 4$.

(a) Find the pdf of X.
(b) Find E(X).
(c) Find P(1 < X < 2) and the median, to 3 significant figures.

(a) $f(x) = F'(x) = \frac{1}{64} x^3$ for $0 \leq x \leq 4$

(b) $E(X) = \int x f(x)\, dx$ from 0 to 4 $= \frac{16}{5}$

(c) $P = F(2) - F(1) = 0.05859$

Median m: $F(m) = 1/2$, so $m = 4 \times (1/2)^{1/4}$

$m = 3.364$

Answer: (a) $f(x) = \frac{1}{64} x^3$ (b) $\frac{16}{5}$ (c) 0.0586, median 3.36