Day 353 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Reading a log graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = ax^n$. The graph of lg y against lg x is the straight line shown.
Find a and n, giving a to 3 s.f.

lg y = lg a + n lg x. The intercept is lg a = 0.5, so $a = 10^{0.5} = 3.162$

$n = \text{gradient} = \frac{6.5 - 0.5}{3} = 2$

Answer: a = 3.16, n = 2

2 Simultaneous logarithmic equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations
$\log_2 x - \log_2 y = 1$
$x + y = 30$

$\log_2(x/y) = 1$, so $x = 2^1 y = 2y$
$2y + y = 30$, so $3y = 30$
Answer: x = 20, y = 10

3 Compound interest and savings (GP) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is dropped from a height of 8 m. After each bounce it rises to $\frac{2}{3}$ of its previous height.

(a) Find the total distance it travels before it stops.
(b) After how many bounces does it rise less than 0.1 m?

(a) Down H, then up and down each time: $H + 2Hr + 2Hr^2 + ...$

$= H + \frac{2Hr}{1 - r} = 8 + \frac{2 \times 8 \times \frac{2}{3}}{1 - \frac{2}{3}} = 40.$

(b) Rebound height after n bounces $= 8(\frac{2}{3})^n < 0.1$.

$n > \frac{\log(0.0125)}{\log(0.666667)} = 10.81$, so n = 11.

Answer: (a) 40 m (b) 11 bounces

4 Integrals of the form f'(x)/f(x) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$$\int_0^2 \frac{2x - 2}{x^2 - 2x + 3}\,dx$$

Numerator = × derivative of denominator
Integral = $[\ln(x^2 - 2x + 3)]$
$= (\ln 3 - \ln 3)$
Answer: 0

5 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 400 kg, carrying a passenger of mass 90 kg, is supported by a vertical cable. The lift is moving downwards and accelerating at 2 m s^{-2}.

Find (a) the force the floor exerts on the passenger, (b) the tension in the cable. Take g = 9.8 m s^{-2}.

Take the direction of the acceleration as positive: it is downwards
Passenger: $90 \times 9.8 - R = 90 \times 2$
$R = 90 \times (9.8 - 2) = 702$
Lift and passenger: $490 \times 9.8 - T = (400 + 90) \times 2$
$T = 490 \times 7.8 = 3822$
Answer: (a) 702 N (b) 3822 N

Day 353 **A Level Maths: S Tier 10-a-day: Model Solutions**

6 Symmetric limits of a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(97, 121^2)$. Find the value k such that $P(97 - k < X < 97 + k) = 0.60$.
Give k to 3 significant figures.

The interval is symmetrical about the mean, so each tail has probability 0.200

$P(X < 97 - k) = 0.200$, so $\frac{-k}{11} = -0.8416$

$k = 11 \times 0.8416 = 9.258$

Answer: k = 9.26

7 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x + 2| \leq |-4x + 5|$.

Square both sides: $(2x + 2)^2 \leq (-4x + 5)^2$.
$(6x - 3)(-2x + 7) \leq 0$.

Critical values $x = \frac{1}{2}$ and $x = \frac{7}{2}$.

The quadratic has negative coefficient of x^2, so choose the outer region.

Answer: $x \leq \frac{1}{2}$ or $x \geq \frac{7}{2}$

8 Hinged rod supported by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (2 m, 40 N) is hinged at A to a vertical wall and held horizontal by a string BC, C on the wall above A, with $\tan \alpha = \frac{4}{3}$, where α is the angle between string and rod. A 40 N particle hangs at P, AP = 0.5 m.

Find (a) the tension, (b) the magnitude and direction of the hinge force (3 s.f. and 1 d.p.).

$\sin \alpha = \frac{4}{5}$, $\cos \alpha = \frac{3}{5}$. Moments about A: $T \sin \alpha \times 2 = 40 \times 1 + 40 \times 0.5$

$T \times \frac{4}{5} \times 2 = 60$, so $T = 37.5$

Horizontal: $H = T \cos \alpha = 22.5$. Vertical: $V + T \sin \alpha = 40 + 40$, so $V = 50$

Force $= \sqrt{(22.5^2 + 50^2)} = 54.8$, angle $= \tan^{-1}(\frac{50}{22.5}) = 65.8°$ above the horizontal

Answer: (a) 37.5 N (b) 54.8 N at 65.8° above the horizontal (away from the wall)

9 Geometric series: least n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 4 and common ratio $\frac{3}{2}$.

Find the least n for which $S_n > 55841$.

$$S_n = \frac{4(\frac{3}{2}^n - 1)}{\frac{1}{2}} > 55841.$$

$\frac{3}{2}^n > 6981.12$.

$n > \frac{\log(6981)}{\log(1.5)} = 21.829$.

So the least n is 22.

Answer: n = 22

10 Perpendicular from a point to a line (3D) (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ -10 \\ 2 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$.

The point P has coordinates $(5, -5, -5)$.

Find the coordinates of the foot F of the perpendicular from P to l, and the exact distance PF.

F = (a + td) for some t, so $\mathbf{FP} = \begin{pmatrix} 4 \\ 5 \\ -7 \end{pmatrix} - t\begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$

$\mathbf{FP} \cdot \mathbf{d} = 0$: $44 - t(22) = 0$, so $t = 2$

F = $(5, -4, -4)$

$\mathbf{PF} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$, so PF = $\sqrt{2}$

Answer: F = $(5, -4, -4)$, PF = $\sqrt{2}$