Day 351 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ax^n$ passes through (9, 324) and (16, 768).
Find the values of a and n.

$324 = a \times 9^n$ and $768 = a \times 16^n$

Divide: $\left(\frac{16}{9}\right)^n = \frac{64}{27}$ so $n = \frac{3}{2}$

$a = 324 \div 9^{\frac{3}{2}} = 12$

Answer: a = 12, n = $\frac{3}{2}$

2 Coded regression line (AQA, Edexcel, OCR, MEI)

Data pairs (x, y) are coded by $u = x - 3$ and $v = \frac{y}{2}$. The regression line of v on u is $v = 10 + 1.1u$.

(a) Find the regression line of y on x in the form $y = a + bx$.
(b) Use it to estimate y when x = 9.

Substitute $u = x - 3$ and $v = y/2$: $\frac{y}{2} = 10 + 1.1(x - 3)$

$y = 2[10 + 1.1x - 3.3] = 2 \times 6.7 + 2 \times 1.1x$
$y = 13.4 + 2.2x$
At x = 9: $y = 13.4 + 2.2 \times 9 = 33.2$
Answer: (a) y = 13.4 + 2.2x (b) 33.2

3 Transforming a graph: images of points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows y = f(x), made of three straight line segments. Find the coordinates of the images of A, B, C and D on the graph of
$y = -f(x - 1) - 1$.

Inside f: replace x by $x - 1$, so x-coordinates become $(x + 1) \div 1$.
Outside: y-coordinates become $-1 \times y - 1$.
A: (−5, −5) B: (1, 2)
C: (5, −5) D: (7, −4)
Answer: A′(−5, −5), B′(1, 2), C′(5, −5), D′(7, −4)

4 sec, cosec and tan/cot quadratics (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $6\text{cosec}^2\theta - 22\cot\theta + 6 = 0$
for $0 \leq \theta < 360°$, giving answers to 1 d.p.

Use $\text{cosec}^2\theta \equiv 1 + \cot^2\theta$: $6\cot^2\theta - 22\cot\theta + 12 = 0$
$2(3\cot\theta - 2)(\cot\theta - 3) = 0$

$\cot\theta = \frac{2}{3}$ or $\cot\theta = 3$

$\tan\theta = \frac{3}{2}$ or $\frac{1}{3}$, so $\theta = 56.3°$ and $18.4°$ $(+180°)$

Answer: θ = 18.4°, 56.3°, 198.4°, 236.3°

5 Hinged rod supported by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (4 m, 40 N) is hinged at A to a vertical wall and held horizontal by a string BC, C on the wall above A, with $\tan\alpha = \frac{4}{3}$, where α is the angle between string and rod. A 40 N particle hangs at P, AP = 3 m.

Find (a) the tension, (b) the magnitude and direction of the hinge force (3 s.f. and 1 d.p.).

$\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$. Moments about A: $T \sin\alpha \times 4 = 40 \times 2 + 40 \times 3$

$T \times \frac{4}{5} \times 4 = 200$, so T = 62.5

Horizontal: $H = T\cos\alpha = 37.5$. Vertical: $V + T\sin\alpha = 40 + 40$, so V = 30

Force $= \sqrt{(37.5^2 + 30^2)} = 48.0$, angle $= \tan^{-1}\left(\frac{30}{37.5}\right) = 38.7°$ above the horizontal

Answer: (a) 62.5 N (b) 48.0 N at 38.7° above the horizontal (away from the wall)

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6 Addition formulae with exact values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is acute and B is acute. $\sin A = \frac{7}{25}$ and $\sin B = \frac{12}{13}$.

Find the exact values of $\tan(A - B)$ and $\cos(A + B)$.

$\cos A = \frac{24}{25}$, $\sin A = \frac{7}{25}$; $\sin B = \frac{12}{13}$, $\cos B = \frac{5}{13}$

$\tan(A - B) = \frac{\sin(A - B)}{\cos(A - B)} = \frac{-\frac{253}{325}}{\frac{204}{325}} = -\frac{253}{204}$

$\cos(A + B) = \cos A \cos B - \sin A \sin B = \frac{36}{325}$

Answer: $\tan(A - B) = -\frac{253}{204}$, $\cos(A + B) = \frac{36}{325}$

7 Compound interest and savings (GP) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each year Jo pays £400 into an account on 1 January.
Interest of 5% is added each 31 December.
Find the first year-end at which the balance exceeds £5,000.

After n years: $400(1.05) + 400(1.05^2) + ... + 400(1.05^n)$.

GP: $a = 400 \times 1.05$, $r = 1.05$: $S_n = \frac{400 \times 1.05\,(1.05^n - 1)}{0.05}$.

$S_n > 5000 \Rightarrow 1.05^n > 1.5952 \Rightarrow n > 9.572$.

So n = 10 years.

Answer: after 10 years

8 Binomial expansion (integer powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of $(3x^3 + \frac{2}{x^2})^{10}$.

General term: $C(10,r)(3x^3)^{10-r}(\frac{2}{x^2})^r$.

Power of x: $3(10 - r) - 2r = 0$, so r = 6.

Term = $C(10,6) \times 3^4 \times (2)^6 = 1088640$.

Answer: 1088640

9 Displacement of the form R sin(ωt + φ) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a line. Its displacement from a fixed point O at time t seconds is s metres, where $s = 8 \sin t + 15 \cos t$.

Find (a) the greatest distance of P from O, (b) the greatest speed of P, (c) the first time P is at its greatest distance from O (3 s.f.).

Write $s = R \sin(t + \varphi)$ with $R \cos \varphi = 8$ and $R \sin \varphi = 15$

$R = \sqrt{(8^2 + 15^2)} = 17$ and $\tan \varphi = \frac{15}{8}$, so $\varphi = 1.081$ rad

Greatest distance = R = 17

$v = \frac{ds}{dt} = 17 \times 1 \cos(t + \varphi)$, so greatest speed = 17

s is greatest when $t + \varphi = \frac{\pi}{2}$: $t = (1.5708 - 1.081) \div 1$

Answer: (a) 17 m (b) 17 m s^{-1} (c) 0.490 s

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10 Critical region, significance level and Type II error (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A test of H_0: p = 0.1 against H_1: p > 0.1 uses a sample of 50 at the 5% level.

(a) Find the critical region for X, the number of successes.

(b) Find the actual significance level.

(c) Find P(Type II error) when in fact p = 0.2.

X ~ B(50, 0.1). P(X ≥ 10) = 0.0245 and P(X ≥ 9) = 0.0579

The first is at most 0.05 and the second is not, so the critical region is X ≥ 10

Actual significance level = P(X ≥ 10) = 0.0245

Type II error: H_0 is not rejected, so X ≤ 9, with X ~ B(50, 0.2)

P(X ≤ 9) = 0.444

Answer: (a) X ≥ 10 (b) 0.0245 (c) 0.444