Day 350 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Range and greatest height to speed and angle (AQA, Edexcel, OCR, MEI, Eduqas)

A golf ball is hit from a point O on horizontal ground. It lands 96 m from O, and reaches a maximum height of 18 m above the ground.

Find the angle of projection and the speed of projection, to 3 s.f. Take g = 9.8 m s^{-2}.

Vertical: $0 = u_y^2 - 2 \times 9.8 \times 18$, so $u_y = 18.78$

Time of flight = $\frac{2u_y}{9.8} = 3.833$ s, so $u_x = \frac{96}{3.833} = 25.04$

$\tan\theta = \frac{u_y}{u_x} = 0.7500$, so $\theta = 36.9°$

Speed = $\sqrt{(u_x^2 + u_y^2)} = 31.3$

Answer: θ = 36.9°, speed 31.3 m s^{-1}

2 Simultaneous logarithmic equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations

$\lg x - \lg y = 1$

$x + y = 121$

$\lg(x/y) = 1$, so $x = 10^1 y = 10y$

$10y + y = 121$, so $11y = 121$

Answer: x = 110, y = 11

3 Surds: rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-1 - 5\sqrt{6}}{2 + \sqrt{6}}$ in the form $p + q\sqrt{6}$, where p and q are rational numbers.

Multiply top and bottom by the conjugate $(2 - \sqrt{6})$.

Numerator: $(-1 - 5\sqrt{6})(2 - \sqrt{6}) = 28 - 9\sqrt{6}$.

Denominator: $2^2 - 1^2(6) = -2$.

So the fraction = $\frac{28 - 9\sqrt{6}}{-2}$.

Answer: $-14 + \frac{9}{2}\sqrt{6}$

4 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (3 kg) lies on a rough table with $\mu = 0.2$, and is joined by a light string over a smooth pulley (see diagram) to particle B (3 kg) on a smooth slope, $\sin\alpha = \frac{4}{5}$.

Released from rest, the string is cut when B has moved 3 m. Find the further distance A slides before stopping.

A: $T - 0.2 \times 3 \times 9.8 = 3a$. B: $3 \times 9.8 \times \frac{4}{5} - T = 3a$

Adding: $9.8(3 \times \frac{4}{5} - 0.2 \times 3) = 6a$, so $a = 2.94$

$T = 3(2.94 + 0.2 \times 9.8) = 14.7$

When cut: $v^2 = 2 \times 2.94 \times 3 = 17.64$. Then A decelerates at $\mu g = 1.96$

Further distance = $\frac{17.64}{2 \times 1.96} = 4.5$

Answer: 4.5 m

5 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both

$x^2 - x - 30 > 0$ and $2x + 6 \leq 6$.

Quadratic: $(x + 5)(x - 6) > 0$ gives $x < -5$ or $x > 6$.

Linear: $2x + 6 \leq 6$ gives $x \leq 0$.

Both must hold: take the overlap of the two sets.

Answer: x < −5

6 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of a apple is N(42, 5^2) g. A apple is 'heavy' if it exceeds 48.
(a) Find the probability that a apple is heavy.
(b) 18 apples are chosen at random. Find the probability that at least 4 are heavy.

(a) P(X > 48) = P(Z > 1.200) = 0.1151
(b) Let Y = number heavy, Y ~ B(18, 0.1151)
P(Y ≥ 4) = 1 − P(Y ≤ 3) = 0.145
Answer: (a) 0.115 (b) 0.145

7 Sums of inverse tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B are acute angles with $\tan A = \frac{1}{10}$ and $\tan B = \frac{9}{11}$.

Find tan(A + B) and hence show that $A + B = \frac{\pi}{4}$.

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{10} + \frac{9}{11}}{1 - \frac{9}{110}} = \frac{\frac{101}{110}}{\frac{101}{110}} = 1$$

A + B is acute, so A + B = π/4

Answer: tan(A + B) = 1, A + B = $\frac{\pi}{4}$

8 Tree diagram with an unknown probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows two games played in turn.

Given the second is a W, P(first was a W) = $\frac{1}{4}$. Find x.

P(W then W) = 0.2x, P(second W) = 0.2x + 0.4(1 − x)

$$\text{P(first W | second W)} = \frac{0.2x}{0.2x + 0.4(1 - x)} = \frac{1}{4}$$

$0.2x = \frac{1}{4}(0.2x + 0.4 - 0.4x)$

Solve the linear equation for x: $x = \frac{2}{5}$

Answer: x = $\frac{2}{5}$

9 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$$\int_1^e \frac{\ln x}{x^2}\,dx$$

Let u = ln x and dv/dx = x^{-2}, so du/dx = $\frac{1}{x}$, v = $-\frac{1}{x}$

Integral = $[-\frac{\ln x}{x}] + \int \frac{1}{x^2}\,dx$

Answer: $1 - 2e^{-1}$

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10 Quadratic sequence from recurrence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$u_{n+1} = u_n + n + 2$, $u_1 = -4$.

(a) Show that $u_n = an^2 + bn + c$ and find a, b, c.

(b) Find the least n for which $u_n > 1453$.

$$u_n = u_1 + \sum_{k=1}^{n-1}(k+2) = -4 + 1 \times \frac{(n-1)n}{2} + 2(n-1).$$

$$= \frac{1}{2}n^2 + \frac{3}{2}n - 6.$$

So $a = \frac{1}{2}$, $b = \frac{3}{2}$, $c = -6$.

$u_{52} = 1424 \leq 1453$ and $u_{53} = 1478 > 1453$.

Answer: (a) $a = \frac{1}{2}$, $b = \frac{3}{2}$, $c = -6$ (b) $n = 53$