Day 348 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Trapezium rule and over/under estimates (AQA, Edexcel, OCR, MEI, Eduqas (trapezium rule))

Use the trapezium rule with 5 strips to estimate

$\int_1^3 \frac{1}{1+x^2}\,dx$, to 4 s.f.

Is it an overestimate or an underestimate? Explain.

$h = \frac{2}{5}$; ordinates at $x = 1, \frac{7}{5}, \frac{9}{5}, \frac{11}{5}, \ldots, 3$

Sum $= \frac{h}{2}[y_0 + y_5 + 2(y_1 + \ldots + y_4)] = 0.4695$

$f''(x) > 0$ on the interval, so the curve is convex and the trapezia lie above the curve

Answer: 0.4695; overestimate

2 Coded data: mean, standard deviation, Σx² (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 40 observations x, $\Sigma(x - 40) = 160$ and $\Sigma(x - 40)^2 = 1800$.

(a) Find the mean of x.

(b) Find the standard deviation of x.

(c) Find the value of Σx^2.

Mean of $(x - 40) = \frac{160}{40} = 4$, so mean of $x = 44$

Variance $= \frac{1800}{40} - (4)^2 = 29$

$\sigma = \sqrt{29}$

$\Sigma x^2 = \Sigma(x - 40)^2 + 80\,\Sigma(x - 40) + 40 \times 40^2$

$= 1800 + 12800 + 64000$

Answer: (a) 44 (b) 5.39 (c) 78600

3 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a point O on horizontal ground with speed 24.5 m s^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Find (a) the time of flight, (b) the range, (c) the greatest height. Take g = 9.8 m s^{-2}.

Components: horizontal 14.7, vertical 19.6 (as $\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$)

Time of flight: $0 = 19.6\,t - 4.9t^2$, so $t = \frac{19.6}{4.9} = 4$

Range $= 14.7 \times 4 = 58.8$

Greatest height: $0 = 19.6^2 - 2 \times 9.8 \times H$, so $H = \frac{384.16}{19.6} = 19.6$

Answer: (a) 4 s (b) 58.8 m (c) 19.6 m

4 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 21 and upper quartile 37. An outlier is a value more than 2 × IQR beyond the nearest quartile.

(a) Find the range of values that are not outliers.

(b) 12 observations have mean 31. The largest is 79, an outlier, and it is removed. Find the new mean.

IQR = 37 − 21 = 16, so 2 × IQR = 32

Lower limit = 21 − 32 = −11, upper limit = 37 + 32 = 69

$\Sigma x = 12 \times 31 = 372$, so without 79: $\Sigma x = 293$

New mean $= \frac{293}{11}$

Answer: (a) −11 to 69 (b) 26.64

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5 Arithmetic series from a sum formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The sum of the first n terms of a sequence is
$S_n = n^2 + 4n$.
(a) Find an expression for u_n.
(b) Find the least n for which $u_n > 158$.

$u_n = S_n - S_{n-1} = n^2 + 4n - [(n-1)^2 + 4(n-1)]$
$= 2n + 3$.
$u_1 = S_1 = 5$ also fits, so $u_n = 2n + 3$ for all n.

$2n + 3 > 158 \Rightarrow n > \frac{155}{2}$, so n = 78.

Answer: (a) $u_n = 2n + 3$ (b) n = 78

6 Addition formulae with exact values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is acute and B is acute. $\sin A = \frac{15}{17}$ and $\sin B = \frac{7}{25}$.
Find the exact values of sin(A + B) and sin(A − B).

$\cos A = \frac{8}{17}$, $\sin A = \frac{15}{17}$; $\sin B = \frac{7}{25}$, $\cos B = \frac{24}{25}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B = \frac{416}{425}$

$\sin(A - B) = \sin A \cos B - \cos A \sin B = \frac{304}{425}$

Answer: $\sin(A + B) = \frac{416}{425}$, $\sin(A - B) = \frac{304}{425}$

7 Transformations of y = x^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe a sequence of two transformations that maps the curve $y = x^2$ onto the curve $y = 3x^2 + 6x + 7$.

Complete the square: $y = 3(x + 1)^2 + 4$.
Step 1: $y = x^2 \rightarrow y = 3x^2$: a stretch, scale factor 3, parallel to the y-axis.
Step 2: replace x by (x + 1) and add 4: translation by $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

Answer: a stretch, scale factor 3, parallel to the y-axis, then translation $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$

8 Volumes of revolution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region bounded by $y = \sin x + 2\cos x$, the x-axis and the lines $x = 0$ and $x = \pi/2$ is rotated through 2π about the x-axis.
Find the exact volume of the solid formed.

$V = \pi \int_0^{\pi/2} y^2 \, dx = \pi \int_0^{\pi/2} (\sin x + 2\cos x)^2 \, dx$
Simplify the integrand, then integrate
$V = \pi \times (2 + \frac{5}{4}\pi)$

Answer: $\frac{1}{4}\pi(8 + 5\pi)$

9 Simultaneous equations (linear and quadratic) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve simultaneously:

$x + y = 4$

$2x^2 - 2xy - y^2 = 32$

From the first equation $x = 4 - y$.

Substitute into the second equation and collect terms:

this gives $3y^2 - 24y = 0$.

$y = 0$ or $y = 8$, giving $x = 4$ or $x = -4$.

Answer: (x, y) = (4, 0) and (−4, 8)

10 Projectile from a cliff (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 29.4 m s^{-1} from the top of a vertical cliff of height 19.6 m, and lands on horizontal ground.

Find (a) the time of flight and the distance from the cliff, (b) the speed and direction on landing. Take $g = 9.8$ m s^{-2}.

Vertically (up positive): $-19.6 = -4.9t^2$, so $t^2 = 4$

$t = 2$ s (the positive root)

Horizontal distance = $29.4 \times 2 = 58.8$ m

On landing: $v_x = 29.4$, $v_y = 0 - 9.8 \times 2 = -19.6$

speed = $\sqrt{(29.4^2 + 19.6^2)} = 35.3$, tan angle = $\frac{19.6}{29.4}$, angle = 33.7° below the horizontal

Answer: (a) 2 s, 58.8 m (b) 35.3 m s^{-1} at 33.7° below the horizontal