Day 347 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 500 kg, carrying a passenger of mass 85 kg, is supported by a vertical cable. The lift is moving upwards and accelerating at 1.2 m s^{-2}.

Find (a) the force the floor exerts on the passenger, (b) the tension in the cable. Take g = 9.8 m s^{-2}.

Take the direction of the acceleration as positive: it is upwards

Passenger: $R - 85 \times 9.8 = 85 \times 1.2$

$R = 85 \times (9.8 + 1.2) = 935$

Lift and passenger: $T - 585 \times 9.8 = (500 + 85) \times 1.2$

$T = 585 \times 11 = 6435$

Answer: (a) 935 N (b) 6435 N

2 Shaded normal curve and the standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The pdf of $X \sim N(37, \sigma^2)$ is shown; the shaded area is symmetrical.

(a) Find σ. (b) Find $P(X > 54)$.

The area 0.9 leaves 0.100 in total in the tails, 0.050 in each

$P(X < 28) = 0.050$ so $\frac{28 - 37}{\sigma} = -1.6449$

$\sigma = \frac{9}{1.6449} = 5.472$

$P(X > 54) = P(Z > 3.107) = 0.000945$

Answer: (a) 5.47 (b) 0.000945

3 Factor theorem and cubic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + ax^2 + bx + 24$, where a and b are constants.

(x + 2) is a factor of f(x), and f(x) leaves remainder −60 when divided by (x + 3).

Find a and b, and hence solve f(x) = 0.

$f(-2) = 0$: $-16 + 4a - 2b + 24 = 0$, so $4a - 2b + 8 = 0$.

$f(-3) = -60$: $9a - 3b - 30 = -60$.

Solving: $a = -6$, $b = -8$.

$f(x) = 2x^3 - 6x^2 - 8x + 24 = 2(x + 2)(x - 2)(x - 3)$

Answer: a = −6, b = −8; x = −2, 2, 3

4 Projectile from a cliff (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 19.6 m s^{-1} from the top of a vertical cliff of height 78.4 m, and lands on horizontal ground.

Find (a) the time of flight and the distance from the cliff, (b) the speed and direction on landing. Take g = 9.8 m s^{-2}.

Vertically (up positive): $-78.4 = -4.9t^2$, so $t^2 = 16$

t = 4 s (the positive root)

Horizontal distance = 19.6 × 4 = 78.4 m

On landing: $v_x = 19.6$, $v_y = 0 - 9.8 \times 4 = -39.2$

speed = $\sqrt{(19.6^2 + 39.2^2)} = 43.8$, tan angle = $\frac{39.2}{19.6}$, angle = 63.4° below the horizontal

Answer: (a) 4 s, 78.4 m (b) 43.8 m s^{-1} at 63.4° below the horizontal

5 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of

$\tan(\tan^{-1}(\frac{4}{7}) + \tan^{-1}(2))$.

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\frac{4}{7} + 2}{1 - \frac{4}{7} \times 2} = \frac{\frac{18}{7}}{-\frac{1}{7}}$

Answer: −18

6 Least number of trials (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each seed is faulty with probability 0.12, independently.
Find the least number n of seeds that must be tested so that the probability of finding at least two faulty is greater than 0.95.

Let X ~ B(n, 0.12). $P(X \geq 2) = 1 - 0.88^n - n(0.12)(0.88)^{n-1}$
This has no simple algebraic solution, so test values of n
n = 37: $P(X \geq 2) = 0.9466$, which is not greater than 0.95
n = 38: $P(X \geq 2) = 0.9520$, which is greater than 0.95
$P(X \geq 2)$ increases with n, so the least value is 38
Answer: n = 38

7 Closest approach of moving points (Edexcel, OCR, MEI, Eduqas, CIE (vectors in kinematics))

Two boats P and Q move so that, t hours after noon, their position vectors in km are
$\mathbf{r}_P = \begin{pmatrix}-3\\5\end{pmatrix} + t\begin{pmatrix}-3\\-2\end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix}-9\\5\end{pmatrix} + t\begin{pmatrix}0\\1\end{pmatrix}$.
Find the time at which they are closest and the exact minimum distance between them.

$\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix}-6\\0\end{pmatrix} + t\begin{pmatrix}3\\3\end{pmatrix}$

Distance$^2 = |\mathbf{r}_Q - \mathbf{r}_P|^2$, minimised when $(\mathbf{r}_Q - \mathbf{r}_P) \cdot \begin{pmatrix}3\\3\end{pmatrix} = 0$

$-18 + t(18) = 0$, so t = 1

Then $\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix}-3\\3\end{pmatrix}$, with length $\sqrt{18}$

Answer: t = 1, minimum distance = $3\sqrt{2}$ km

8 Reciprocal graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve C, its asymptotes (dashed) and a point P. C has equation
$y = \frac{a}{x + p} + q$. Find a, p and q.

Asymptotes x = −3 and y = 2, so p = 3 and q = 2.

$y = \frac{a}{x + 3} + 2$.

P(−4, −2): $-2 = \frac{a}{-1} + 2$, so a = 4.

Answer: a = 4, p = 3, q = 2

9 Area between a curve and a line (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = \frac{5}{x}$ and the line y = 6 − x.
Find the exact area of the shaded region enclosed by the curve and the line.

Intersect: $\frac{5}{x} = 6 - x$ gives $x^2 - 6x + 5 = 0$, so x = 1 and x = 5

Area $= \int_1^5 (6 - x - \frac{5}{x})\,dx = [6x - \frac{x^2}{2} - 5\ln x]$

$= (30 - \frac{25}{2} - 5\ln 5) - (6 - \frac{1}{2} - 5\ln 1)$

Answer: 12 − 5ln 5

10 Recurrence: limit and convergence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$u_{n+1} = -\frac{1}{2}u_n + \frac{27}{2}$, $u_1 = 33$.

(a) Find the limit L of the sequence.

(b) Find the least n such that $|u_n - L| < 0.001$.

(a) At the limit $L = -\frac{1}{2}L + \frac{27}{2}$, so $L(1 - (-\frac{1}{2})) = \frac{27}{2}$.

$L = 9$.

(b) $u_n - L = (u_1 - L)((-\frac{1}{2}))^{n-1} = 24((-\frac{1}{2}))^{n-1}$.

Need $24 \times (\frac{1}{2})^{n-1} < 0.001$, so $n - 1 > 14.551$.

The least n is 16.

Answer: (a) L = 9 (b) n = 16