Day 344 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Binomial: mean and variance given (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A binomial random variable X has mean 12 and variance 9.
(a) Find n and p.
(b) Find P(X = 13), correct to 3 significant figures.

$np = 12$ and $np(1 - p) = 9$

Divide: $1 - p = 9 \div 12 = \frac{3}{4}$, so $p = \frac{1}{4}$

$n = 12 \div \frac{1}{4} = 48$

$P(X = 13) = C(48, 13)\ (\frac{1}{4})^{13}\ (\frac{3}{4})^{35} = 0.122$

Answer: (a) n = 48, p = $\frac{1}{4}$ (b) 0.122

2 Equidistant point and triangle area (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A(−3, 5) and B(0, −4). The point C lies on the x-axis and CA = CB. Find the coordinates of C and the area of triangle ABC.

C = (c, 0): $(c - (-3))^2 + 25 = (c - 0)^2 + 16$.
Expand and solve: $6c = -18$, so $c = -3$.
Area = $\frac{1}{2}|(x_A - c)y_B - (x_B - c)y_A|$

$= \frac{1}{2}|0 \times (-4) - 3 \times 5| = \frac{15}{2}$.

Answer: C(−3, 0); area $\frac{15}{2}$

3 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves with constant acceleration. At t = 0 seconds P is at $(4\mathbf{i} + 3\mathbf{j})$ m with velocity $(11\mathbf{i} + 5\mathbf{j})$ m s^{-1}. At t = 6, P has velocity $(-13\mathbf{i} - 19\mathbf{j})$ m s^{-1}.
Find (a) the acceleration of P, (b) the time at which P moves parallel to $(\mathbf{i} - \mathbf{j})$, (c) the position vector of P at that time.

$\mathbf{a} = \frac{\mathbf{v} - \mathbf{u}}{t} = \frac{(-13\mathbf{i} - 19\mathbf{j}) - (11\mathbf{i} + 5\mathbf{j})}{6} = (-4\mathbf{i} - 4\mathbf{j})$

$\mathbf{v} = (11\mathbf{i} + 5\mathbf{j}) + t(-4\mathbf{i} - 4\mathbf{j})$
Parallel to $(\mathbf{i} - \mathbf{j})$: $(-4t + 11) \times -1 = (-4t + 5) \times 1$, giving $t = 2$
$\mathbf{r} = (4\mathbf{i} + 3\mathbf{j}) + 2(11\mathbf{i} + 5\mathbf{j}) + \frac{1}{2} \times 2^2\ (-4\mathbf{i} - 4\mathbf{j}) = (18\mathbf{i} + 5\mathbf{j})$
Answer: (a) $(-4\mathbf{i} - 4\mathbf{j})$ m s^{-2} (b) t = 2 (c) $(18\mathbf{i} + 5\mathbf{j})$ m

4 sec, cosec and tan/cot quadratics (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sec^2\theta + 5\tan\theta + 5 = 0$
for $0 \leq \theta < 360°$, giving answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $\tan^2\theta + 5\tan\theta + 6 = 0$
$(\tan\theta + 2)(\tan\theta + 3) = 0$
$\tan\theta = -2$ or $\tan\theta = -3$
$\theta = 116.6°$ and $108.4°$ (+180°)
Answer: θ = 108.4°, 116.6°, 288.4°, 296.6°

5 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both
$x^2 - 3x - 28 \geq 0$ and $2x - 1 \geq 1$.

Quadratic: $(x + 4)(x - 7) \geq 0$ gives $x \leq -4$ or $x \geq 7$.
Linear: $2x - 1 \geq 1$ gives $x \geq 1$.
Both must hold: take the overlap of the two sets.
Answer: x ≥ 7

6 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2\cos t$, $y = 6\sin t$.
Find a cartesian equation of the curve, with no fractions.

$\cos t = \frac{x}{2}$, $\sin t = \frac{y}{6}$

Use $\cos^2 t + \sin^2 t = 1$: $\frac{x^2}{4} + \frac{y^2}{36} = 1$

Multiply by 144

Answer: $36x^2 + 4y^2 = 144$

7 Range and domain by completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$g(x) = \sqrt{-x^2 - x + 6}$.
Find the largest possible domain of g, and the corresponding range.

Need $-x^2 - x + 6 \geq 0$, i.e. $(x + 3)(x - 2) \leq 0$.
Domain: $-3 \leq x \leq 2$.

Complete the square: $-x^2 - x + 6 = \frac{25}{4} - (x + \frac{1}{2})^2$.

Greatest value of $g = \sqrt{\frac{25}{4}} = \frac{5}{2}$, and least value is 0.

Answer: $-3 \leq x \leq 2$; $0 \leq g(x) \leq \frac{5}{2}$

8 Triangle in 3D: angle and area (CIE (9709); Further Maths on other boards)

The points A, B, C have coordinates (4, 4, −2), (1, −2, −2), (2, −3, −2).
Find angle ABC to 1 d.p., and the exact area of triangle ABC.

$\mathrm{BA} = \begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix}$, $\mathrm{BC} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$; $\mathrm{BA} \cdot \mathrm{BC} = -3$

$|\mathrm{BA}| = \sqrt{45}$, $|\mathrm{BC}| = \sqrt{2}$

$\cos \mathrm{ABC} = \frac{-3}{\sqrt{45}\sqrt{2}} = -0.3162$, so ABC = 108.4°

$|\mathrm{BA} \times \mathrm{BC}| = \left|\begin{pmatrix} 0 \\ 0 \\ -9 \end{pmatrix}\right| = \sqrt{81}$

Area $= \frac{1}{2}|\mathrm{BA} \times \mathrm{BC}|$

Answer: ABC = 108.4°, area = $\frac{9}{2}$

9 Critical region, significance level and Type II error (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A test of H_0: $p = 0.5$ against H_1: $p < 0.5$ uses a sample of 30 at the 1% level.
(a) Find the critical region for X, the number of successes.
(b) Find the actual significance level.
(c) Find P(Type II error) when in fact $p = 0.35$.

$X \sim B(30, 0.5)$. $P(X \leq 8) = 0.00806$ and $P(X \leq 9) = 0.0214$
The first is at most 0.01 and the second is not, so the critical region is $X \leq 8$
Actual significance level = $P(X \leq 8) = 0.00806$
Type II error: H_0 is not rejected, so $X \geq 9$, with $X \sim B(30, 0.35)$
$P(X \geq 9) = 1 - P(X \leq 8) = 0.775$
Answer: (a) $X \leq 8$ (b) 0.00806 (c) 0.775

10 Hinged rod supported by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (4 m, 80 N) is hinged at A to a vertical wall and held horizontal by a string BC, C on the wall above A, with $\tan\alpha = \frac{5}{12}$, where α is the angle between string and rod. A 20 N particle hangs at P, AP = 3 m.

Find (a) the tension, (b) the magnitude and direction of the hinge force (3 s.f. and 1 d.p.).

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$. Moments about A: $T\sin\alpha \times 4 = 80 \times 2 + 20 \times 3$

$T \times \frac{5}{13} \times 4 = 220$, so $T = 143$

Horizontal: $H = T\cos\alpha = 132$. Vertical: $V + T\sin\alpha = 80 + 20$, so $V = 45$

Force $= \sqrt{(132^2 + 45^2)} = 139$, angle $= \tan^{-1}(\frac{45}{132}) = 18.8°$ above the horizontal

Answer: (a) 143 N (b) 139 N at 18.8° above the horizontal (away from the wall)