Day 342 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Conditions on the roots of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The roots of $x^2 + kx + 60 = 0$ differ by 4.
Find the possible values of k.

Let the roots be α and $\alpha + 4$.

Product: $\alpha(\alpha + 4) = 60$, so $\alpha^2 + 4\alpha - 60 = 0$.
$(\alpha - 6)(\alpha + 10) = 0$, so $\alpha = 6$ or $\alpha = -10$.
Sum: $2\alpha + 4 = -k$, so $k = -(2\alpha + 4) = -16$ or 16.
Answer: $k = \pm 16$

2 Dropped and projected particles meet (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Ball A is projected vertically upwards from ground level with speed 27 m s^{-1}. At the same instant, ball B is released from rest 108 m directly above A.
The balls collide. Find (a) the height above the ground at which they collide, (b) the velocity of A just before the collision.
Take $g = 9.8$ m s^{-2}.

Relative to B, A moves upwards at the constant speed 27 m s^{-1} (both have acceleration g)
So the balls meet when $27\,t = 108$, giving $t = 4$
Height of A: $s = 27 \times 4 - \frac{1}{2} \times 9.8 \times 4^2 = 29.6$
$v = u - gt = 27 - 9.8 \times 4 = -12.2$
Answer: (a) 29.6 m (b) 12.2 m s^{-1} downwards

3 Arithmetic series modelling (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first row of a theatre has 11 seats. Each row has 2 more seats than the row in front. Find
(a) the number of seats in row 30,
(b) the least number of rows for at least 1295 seats.

Arithmetic: $a = 11$, $d = 2$.
(a) $u_{30} = 11 + 29 \times 2 = 69$.
(b) $S_n = \frac{n}{2}(22 + 2n - 2) = \frac{n(2n + 20)}{2} \geq 1295$.
$S_{31} = 1271 < 1295$, $S_{32} = 1344 \geq 1295$.
Answer: (a) 69 (b) 32 rows

4 Newton cooling model (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A drink cools according to $\theta = 18 + Ae^{-kt}$, where θ °C is the temperature after t minutes. Initially $\theta = 90$, and after 8 minutes $\theta = 54$.
Find A and k, then find when $\theta = 29$ (3 s.f.).

$t = 0$: $18 + A = 90$ so $A = 72$

$t = 8$: $18 + 72e^{-8k} = 54$ so $e^{-8k} = \frac{1}{2}$

$k = \frac{1}{8} \ln 2 = 0.0866$

$72\,e^{-kt} = 11$ so $t = \frac{1}{k} \ln \frac{72}{11}$

$t = 21.7$ minutes
Answer: A = 72, k = 0.0866, t = 21.7 minutes

5 Replacing an observation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

12 recorded lengths (cm) have mean 60 and standard deviation 5.
One value was recorded as 52 but should have been 46.
Find the correct mean and the correct standard deviation.

$\Sigma x = 12 \times 60 = 720$, so corrected $\Sigma x = 720 - 52 + 46 = 714$

Correct mean $= \frac{714}{12} = 59.50$

$\Sigma x^2 = 12(25 + 3600) = 43500$, corrected $\Sigma x^2 = 43500 - 2704 + 2116 = 42912$

Variance $= \frac{42912}{12} - 59.500^2 = 35.750$, $\sigma = 5.979$

Answer: mean 59.5, standard deviation 5.98

6 Implicit differentiation: tangent or normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x e^y + y = 4$.
Find the equation of the tangent to C at the point (4, 0),
in the form $ax + by + c = 0$ with integers a, b, c.

Differentiate implicitly with respect to x

At (4, 0): $dy/dx = -\frac{1}{5}$

$y - 0 = (-\frac{1}{5})(x - 4)$, then clear fractions

Answer: $x + 5y - 4 = 0$

7 Independence from a Venn diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The Venn diagram shows probabilities; A and B are independent.
Find x and y.

$P(A) = 2x + x = 3x$, $P(B) = x + y$, $P(A \cap B) = x$

Independent: $x = 3x(x + y)$, and $x \neq 0$, so $x + y = \frac{1}{3}$

Total probability: $2x + x + y + 0.35 = 1$, so $2x + (x + y) = 0.65$

$2x = 0.65 - \frac{1}{3} = \frac{19}{60}$

$x = \frac{19}{120}$, $y = \frac{1}{3} - x = \frac{7}{40}$

Answer: $x = \frac{19}{120}$, $y = \frac{7}{40}$

8 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (4 kg) lies on a rough table with $\mu = 0.1$, and is joined by a light string over a smooth pulley (see diagram) to particle B (4 kg) on a smooth slope, $\sin \alpha = \frac{3}{5}$.

Released from rest, the string is cut when B has moved 0.5 m. Find the further distance A slides before stopping.

A: $T - 0.1 \times 4 \times 9.8 = 4a$. B: $4 \times 9.8 \times \frac{3}{5} - T = 4a$

Adding: $9.8(4 \times \frac{3}{5} - 0.1 \times 4) = 8a$, so $a = 2.45$

$T = 4(2.45 + 0.1 \times 9.8) = 13.72$

When cut: $v^2 = 2 \times 2.45 \times 0.5 = 2.45$. Then A decelerates at $\mu g = 0.98$

Further distance $= \frac{2.45}{2 \times 0.98} = 1.25$

Answer: 1.25 m

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9 Remainder theorem (quadratic divisor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when x^7 is divided by $(x - 2)(x + 2)$.

$x^7 = (x - 2)(x + 2)Q(x) + px + q$.
$x = 2$: $128 = 2p + q$.
$x = -2$: $-128 = -2p + q$.
Subtract: $4p = 256$, so $p = 64$ and $q = 0$.
Answer: 64x

10 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas, CIE (small angle))

θ is small and measured in radians. Use small angle approximations to find the value that

$$\frac{\cos 4\theta - \cos 5\theta}{\theta^2}$$

approaches as $\theta \to 0$.

$\cos 4\theta \approx 1 - 8\theta^2$ and $\cos 5\theta \approx 1 - \frac{25}{2}\theta^2$

Difference $\approx \frac{9}{2}\theta^2$, so the expression $\approx \frac{9}{2}$

Answer: $\frac{9}{2}$