Day 341 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Coded data: mean, standard deviation, Σx^2 (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 50 observations x, $\Sigma(x - 100) = 10$ and $\Sigma(x - 100)^2 = 402$.
(a) Find the mean of x.
(b) Find the standard deviation of x.
(c) Find the value of Σx^2.

Mean of $(x - 100) = \frac{10}{50} = 0.2$, so mean of $x = 100.2$

Variance $= \frac{402}{50} - (0.2)^2 = 8$

$\sigma = \sqrt{8}$

$\Sigma x^2 = \Sigma(x - 100)^2 + 200\,\Sigma(x - 100) + 50 \times 100^2$

$= 402 + 2000 + 500000$

Answer: (a) 100.2 (b) 2.83 (c) 502402

2 Double angle equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cos 2\theta + \sin\theta = 0$
for $0 \leq \theta < 2\pi$, giving answers in terms of π.

Use $\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{2}$ or $\sin\theta = 1$

Principal values and symmetry in $[0, 2\pi)$

Answer: $\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

3 Partial fractions (repeated factor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-2x^2 + 30x - 40}{(x + 1)(2x - 4)^2}$ in the form

$\frac{A}{(x + 1)} + \frac{B}{(2x - 4)} + \frac{C}{(2x - 4)^2}$.

$-2x^2 + 30x - 40 \equiv A(2x - 4)^2 + B(x + 1)(2x - 4) + C(x + 1)$.

Put $x = -1$: $-72 = A(-6)^2$, so $A = -2$.

Put $x = 2$: $12 = C(3)$, so $C = 4$.

Compare x^2: $-2 = 4A + 2B$, so $B = 3$.

Answer: A = −2, B = 3, C = 4

4 Inverse trigonometric values (Edexcel, OCR, MEI, Eduqas, CIE (inverse trig))

Find the exact value of

$\tan(\tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{2}{3}))$.

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\frac{1}{7} + \frac{2}{3}}{1 - \frac{1}{7} \times \frac{2}{3}} = \frac{\frac{17}{21}}{\frac{19}{21}}$

Answer: $\frac{17}{19}$

Day 341 **A Level Maths: S Tier 10-a-day: Model Solutions**

5 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 34 and upper quartile 47. An outlier is a value more than 1 × IQR beyond the nearest quartile.
(a) Find the range of values that are not outliers.
(b) 10 observations have mean 42. The largest is 71, an outlier, and it is removed. Find the new mean.

IQR = 47 − 34 = 13, so 1 × IQR = 13
Lower limit = 34 − 13 = 21, upper limit = 47 + 13 = 60
$\Sigma x = 10 \times 42 = 420$, so without 71: $\Sigma x = 349$
New mean = $\frac{349}{9}$

Answer: (a) 21 to 60 (b) 38.78

6 sec, cosec and tan/cot quadratics (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\sec^2\theta + \tan\theta - 3 = 0$
for $0 \leq \theta < 360°$, giving answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $\tan^2\theta + \tan\theta - 2 = 0$
$(\tan\theta - 1)(\tan\theta + 2) = 0$
$\tan\theta = 1$ or $\tan\theta = -2$
$\theta = 45.0°$ and $116.6°$ $(+180°)$
Answer: θ = 45.0°, 116.6°, 225.0°, 296.6°

7 Factor theorem and cubic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 3x^3 + ax^2 + bx - 24$, where a and b are constants.
$(x - 2)$ is a factor of f(x), and f(x) leaves remainder 30 when divided by $(x + 3)$.
Find a and b, and hence solve f(x) = 0.

$f(2) = 0$: $24 + 4a + 2b - 24 = 0$, so $4a + 2b = 0$.
$f(-3) = 30$: $9a - 3b - 105 = 30$.
Solving: $a = 9$, $b = -18$.
$f(x) = 3x^3 + 9x^2 - 18x - 24 = 3(x + 4)(x + 1)(x - 2)$
Answer: a = 9, b = −18; x = −4, −1, 2

8 Non-uniform rod: centre of mass and tilting (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB of length 5 m and mass 12 kg rests on supports at C and D, where AC = 1 m and AD = 4 m, with $R_C = 3 R_D$.
Find (a) the distance of the centre of mass from A. A particle of mass m kg at B puts the rod on the point of tilting about D.
(b) Find m.

$R_C = 3 R_D$. Moments about A: $R_C \times 1 + R_D \times 4 = 12\,g \times AG$
$R_D(3 \times 1 + 4) = 12\,g \times AG$ with $R_D(1 + 3) = 12\,g$, so $AG = \frac{3 \times 1 + 4}{4} = 1.75$
Tilting about D: $R_C = 0$. Moments about D: $m\,g\,(5 - 4) = 12\,g\,(4 - 1.75)$
$m = \frac{12 \times 2.25}{1} = 27$

Answer: (a) 1.75 m (b) 27

Day 341 **A Level Maths: S Tier 10-a-day: Model Solutions**

9 Binomial expansion with partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{3x + 9}{(1 + 2x)(1 - x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) in ascending powers of x up to x^3, stating the range of validity.

(a) $3x + 9 \equiv A(1 - x) + B(1 + 2x)$: $A = 5$, $B = 4$.

$f(x) = \frac{5}{1 + 2x} + \frac{4}{1 - x}$.

(b) $\frac{1}{1 + cx} = 1 - cx + c^2x^2 - c^3x^3 + \ldots$ (use $c = 2$, then $c = -1$).

Coefficients of x^j: $5(-2)^j + 4(1)^j$.

Valid for $|x| < \frac{1}{2}$ (both expansions).

Answer: (a) $A = 5$, $B = 4$ (b) $9 - 6x + 24x^2 - 36x^3 + \ldots$; $|x| < \frac{1}{2}$

10 Interception at constant speed (AQA, Edexcel, OCR, MEI, Eduqas)

At time t = 0 hours, ship A is at the point with position vector $(26\mathbf{i} + 24\mathbf{j})$ km relative to a port O, and moves with constant velocity $(-5\mathbf{i} + 3\mathbf{j})$ km h^{-1}. Ship B leaves O at t = 0 with constant speed 17 km h^{-1} and intercepts A.
Find (a) the time at which B intercepts A, (b) the velocity of B as a vector.

Position of A at time t: $(26\mathbf{i} + 24\mathbf{j}) + t(-5\mathbf{i} + 3\mathbf{j})$. B at time t: t v_B (unknown velocity)
At interception B has travelled 17 t, so $|(26\mathbf{i} + 24\mathbf{j}) + t(-5\mathbf{i} + 3\mathbf{j})| = 17\,t$
Squaring and rearranging: $-255t^2 - 116t + 1252 = 0$, which gives t = 2 or a negative value
t = 2 (the other root is negative since B is faster than A)

A is then at $(26\mathbf{i} + 24\mathbf{j}) + 2(-5\mathbf{i} + 3\mathbf{j}) = (16\mathbf{i} + 30\mathbf{j})$, so $v_B = \frac{1}{2}(16\mathbf{i} + 30\mathbf{j}) = (8\mathbf{i} + 15\mathbf{j})$

Answer: (a) 2 h (b) $(8\mathbf{i} + 15\mathbf{j})$ km h^{-1}