Day 338 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Replacing an observation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

10 recorded times (seconds) have mean 53 and standard deviation 2.
One value was recorded as 48 but should have been 53.
Find the correct mean and the correct standard deviation.

$\Sigma x = 10 \times 53 = 530$, so corrected $\Sigma x = 530 - 48 + 53 = 535$

Correct mean $= \frac{535}{10} = 53.50$

$\Sigma x^2 = 10(4 + 2809) = 28130$, corrected $\Sigma x^2 = 28130 - 2304 + 2809 = 28635$

Variance $= \frac{28635}{10} - 53.500^2 = 1.2500$, $\sigma = 1.118$

Answer: mean 53.5, standard deviation 1.12

2 Related rates of change (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cone with its vertex downwards and semi-vertical angle α, where
$\tan\alpha = \frac{1}{2}$, is filled at 12π cm^{3}s^{-1}. Find the rate at which
the depth h is rising when h = 5 cm. ($V = \frac{1}{3}\pi r^2 h$)

$r = \frac{h}{2}$, so $V = \frac{1}{3}\pi \times \frac{h^2}{4} \times h = \frac{\pi}{12}h^3$

$dV/dt = \frac{\pi}{4}h^2\, dh/dt$, so $12\pi = \frac{\pi}{4} \times 25 \times dh/dt$

$dh/dt = \frac{48}{25}$

Answer: $\frac{48}{25}$ cm s^{-1}

3 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$g(x) = x^2 + 5$. Also $fg(x) = 4x^2 + 15$, where f is
a linear function. (a) Find f(x).
(b) Solve $gf(x) = 54$.

(a) Let $f(x) = ax + b$. $fg(x) = a(x^2 + 5) + b = 4x^2 + 15$.
So $a = 4$ and $20 + b = 15$, giving $b = -5$.
$f(x) = 4x - 5$.
(b) $gf(x) = (4x - 5)^2 + 5 = 54$.
$(4x - 5)^2 = 49$, so $4x - 5 = \pm 7$.
$x = -\frac{1}{2}$ or $x = 3$.

Answer: (a) f(x) = 4x − 5 (b) $x = -\frac{1}{2}$ or x = 3

4 Differentiating a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact gradient of the curve $y = 5^{2x+2}$
at the point where x = 1.

$y = e^{(2x+2)\ln 5}$

$dy/dx = 2 \ln 5 \times 5^{2x+2}$

At x = 1: exponent = 4, so $dy/dx = 2 \ln 5 \times 5^4$

Answer: 1250ln 5

5 Reading a log graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$y = ax^n$. The graph of lg y against lg x is the straight line shown.
Find a and n, giving a to 3 s.f.

lg y = lg a + n lg x. The intercept is lg a = 0.8, so $a = 10^{0.8} = 6.310$

n = gradient $= \frac{6.8 - 0.8}{3} = 2$

Answer: a = 6.31, n = 2

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6 Partial fractions (repeated factor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-2x^2 + 14x - 20}{(x + 3)(x - 1)^2}$ in the form $\frac{A}{(x + 3)} + \frac{B}{(x - 1)} + \frac{C}{(x - 1)^2}$.

$-2x^2 + 14x - 20 \equiv A(x - 1)^2 + B(x + 3)(x - 1) + C(x + 3)$.

Put $x = -3$: $-80 = A(-4)^2$, so $A = -5$.

Put $x = 1$: $-8 = C(4)$, so $C = -2$.

Compare x^2: $-2 = 1A + 1B$, so $B = 3$.

Answer: A = −5, B = 3, C = −2

7 Least force to drag a box (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of weight 50 N is pulled at constant speed along rough horizontal ground ($\mu = 0.75$) by a force P N at angle θ above the horizontal.

(a) Find P when $\tan\theta = \frac{4}{3}$. (b) Find the least possible value of P as θ varies, and the corresponding θ to 1 d.p.

Vertical: $R + P\sin\theta = 50$. Horizontal: $P\cos\theta = \mu R = 0.75(50 - P\sin\theta)$

$P(\cos\theta + 0.75\sin\theta) = 0.75 \times 50$, so $P = 37.5 \div (\cos\theta + 0.75\sin\theta)$

(a) $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$: $P = 37.5 \div 1.2 = 31.25$

(b) P is least when $\cos\theta + 0.75\sin\theta$ is greatest. Its greatest value is $\sqrt{(1 + 0.75^2)} = 1.25$, which occurs when $\tan\theta = 0.75$. Then $P = 37.5 \div 1.25 = 30$ and $\theta = 36.9°$

Answer: (a) 31.25 N (b) 30 N at θ = 36.9°

8 Binomial expansion (integer powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 24 and the coefficient of x^2 is 240. Find n and k.

Coefficient of x: $nk = 24$. Coefficient of x^2: $\frac{n(n - 1)}{2}k^2 = 240$.

Divide the second by $(nk)^2$: $\frac{n - 1}{2n} = \frac{240}{576}$.

$2n \times 240 = 576(n - 1) \Rightarrow n = \frac{576}{96} = 6$.

Then $k = \frac{24}{6} = 4$.

Answer: n = 6, k = 4

9 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population N satisfies $\frac{dN}{dt} = -0.02N^2$, with $N = 50$ when $t = 0$.

Find N when $t = 9$, and the time taken for N to halve.

$\int N^{-2}\,dN = -0.02\int dt$ gives $-\frac{1}{N} = -0.02t + c$

$t = 0$: $c = -\frac{1}{50}$, so $\frac{1}{N} = 0.02t + \frac{1}{50}$

$t = 9$: $\frac{1}{N} = \frac{9}{50} + \frac{1}{50}$; halving: $0.02t = \frac{1}{50}$

Answer: N = 5; t = 1

10 Block on a plank: limiting slip (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A plank of mass 8 kg lies on a smooth horizontal table, with a block of mass 1 kg on it ($\mu = 0.2$ between them). A horizontal force P N acts on the plank.

(a) Find the greatest P for which the block does not slip. (b) Find the accelerations of block and plank when P = 35.28. Take g = 9.8 m s^{-2}.

No slipping: common acceleration $a = \frac{P}{9}$. The block is accelerated only by friction: $F = 1\,a \le \mu mg$

So $a \le \mu g = 1.96$, and $P_{max} = 9 \times 1.96 = 17.64$

$P = 35.28 > 17.64$, so the block slips. Block: $a = \mu g = 1.96$

Plank: $35.28 - 1.96 = 8\,a$, so $a = 4.165$

Answer: (a) 17.64 N (b) block 1.96 m s^{-2}, plank 4.165 m s^{-2}