**Day 337** **A Level Maths: S Tier 10-a-day: Model Solutions**

## 1 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 300 kg, carrying a passenger of mass 90 kg, is supported by a vertical cable. The lift is moving upwards and accelerating at 2.4 m $s^{-2}$.

Find (a) the force the floor exerts on the passenger, (b) the tension in the cable. Take g = 9.8 m $s^{-2}$.

Take the direction of the acceleration as positive: it is upwards

Passenger: $R - 90 \times 9.8 = 90 \times 2.4$

$R = 90 \times (9.8 + 2.4) = 1098$

Lift and passenger: $T - 390 \times 9.8 = (300 + 90) \times 2.4$

$T = 390 \times 12.2 = 4758$

**Answer: (a) 1098 N (b) 4758 N**

## 2 Modelling y = ab^x and y = ax^n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = ax^n$ passes through (1, 6) and (3, 54).

Find the values of a and n.

$6 = a \times 1^n$ and $54 = a \times 3^n$

Divide: $3^n = 9$ so $n = 2$

$a = 6 \div 1^2 = 6$

**Answer: a = 6, n = 2**

## 3 Binomial: finding p from a ratio (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(12, p). It is given that $P(X = 2) = \frac{11}{10} \times P(X = 1)$.

(a) Find the value of p.

(b) Find P(X = 0), correct to 3 significant figures.

$\frac{132}{2} p^2(1 - p)^{10} = \frac{11}{10} \times 12\, p(1 - p)^{11}$

Divide by $p(1 - p)^{10}$: $5.5\, p = \frac{11}{10}(1 - p)$

$(5.5 + \frac{11}{10})p = \frac{11}{10}$

$p = \frac{1}{6}$

$P(X = 0) = (1 - p)^{12} = (\frac{5}{6})^{12} = 0.112$

**Answer: (a) $p = \frac{1}{6}$ (b) 0.112**

## 4 Distance travelled with a change of direction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time t seconds its velocity is v m $s^{-1}$, where

$v = t^2 - 5t + 4$ for $0 \leq t \leq 6$

Find (a) the times at which P is at instantaneous rest, (b) the total distance travelled by P in the first 6 seconds.

$v = (t - 1)(t - 4)$, so v = 0 when t = 1 and t = 4

v > 0 for 0 < t < 1, v < 0 for 1 < t < 4, v > 0 for t > 4: direction changes at each

$s = \int v\, dt = \frac{1}{3}t^3 - 2.5t^2 + 4t$

Distances: 0 to 1: $\frac{11}{6}$; 1 to 4: $\frac{9}{2}$; 4 to 6: $\frac{26}{3}$

Total = $\frac{11}{6} + \frac{9}{2} + \frac{26}{3}$

**Answer: (a) t = 1, t = 4 (b) 15 m**

## 5 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$g(x) = x^2 - 2$. Also $fg(x) = -3x^2$, where f is a linear function. (a) Find f(x).
(b) Solve $gf(x) = 7$.

(a) Let $f(x) = ax + b$. $fg(x) = a(x^2 - 2) + b = -3x^2$.
So $a = -3$ and $6 + b = 0$, giving $b = -6$.
$f(x) = -3x - 6$.
(b) $gf(x) = (-3x - 6)^2 - 2 = 7$.
$(-3x - 6)^2 = 9$, so $-3x - 6 = \pm 3$.
$x = -3$ or $x = -1$.
**Answer: (a) $f(x) = -3x - 6$ (b) $x = -3$ or $x = -1$**

## 6 Alternate turns and sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B take turns, A first, to score; the first to score wins. P(A scores) = $\frac{2}{5}$, P(B scores) = $\frac{1}{6}$ each attempt.
Find P(A wins (a) on attempt 1 or 2, (b) overall).

P(A wins at attempt 1) = $\frac{2}{5}$; to reach attempt 2 both miss first: $(\frac{3}{5})(\frac{5}{6}) = \frac{1}{2}$

(a) $\frac{2}{5} + \frac{1}{2} \times \frac{2}{5} = \frac{3}{5}$

(b) P(A wins) = $\frac{2}{5} + \frac{1}{2} \times \frac{2}{5} + \frac{1}{2}^2 \times \frac{2}{5} + ...$

Geometric: first term $\frac{2}{5}$, common ratio $\frac{1}{2}$

$S = \frac{a}{1 - r} = \frac{\frac{2}{5}}{\frac{1}{2}} = \frac{4}{5}$

**Answer: (a) $\frac{3}{5}$ (b) $\frac{4}{5}$**

## 7 Polynomial inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^4 - x^3 - 15x^2 + 9x + 54 < 0$.

Use the factor theorem to find roots: $-3$, $-2$, $3$.
$f(x) = (x + 3)(x + 2)(x - 3)^2$.
Sign of f in each region (left to right): +, −, +, +.
Take the regions where $f < 0$ (include roots if $\leq$ or $\geq$).
**Answer: $-3 < x < -2$**

## 8 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \frac{1}{21 + 8 \sin\theta + 15 \cos\theta}$
(a) Write $8 \sin\theta + 15 \cos\theta$ as $R \sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.
(b) Find the greatest and least values of $f(\theta)$, and the smallest positive $\theta$ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{8^2 + 15^2} = 17$, $\tan\alpha = \frac{15}{8}$, $\alpha = 61.9°$

Denominator = $21 + 17 \sin(\theta + 61.9°)$, which lies between 4 and 38

Greatest $f(\theta) = \frac{1}{4} = \frac{1}{4}$, when $\sin(\theta + \alpha) = -1$

Least $f(\theta) = \frac{1}{38} = \frac{1}{38}$, when $\sin(\theta + \alpha) = 1$

$\sin(\theta + \alpha) = -1$: $\theta + 61.9° = 270°$, so $\theta = 208.1°$

**Answer: (a) $17 \sin(\theta + 61.9°)$ (b) greatest $\frac{1}{4}$, least $\frac{1}{38}$, $\theta = 208.1°$**

## 9 Line tangent to a hyperbola (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = \frac{2x - 3}{x - 2}$.

(a) Write down the equations of the asymptotes of C.
(b) The line $y = -x + k$ is a tangent to C.
Find the possible values of k.

(a) Vertical: $x = 2$. As $x \to \infty$, $y \to 2$, so $y = 2$.
(b) $2x - 3 = (-x + k)(x - 2)$.
$-x^2 + kx + (-2k + 3) = 0$.
Tangent: $b^2 - 4ac = 0$, which simplifies to $k^2 - 8k + 12 = 0$.
**Answer: (a) $x = 2$, $y = 2$ (b) $k = 2$ or $k = 6$**

## 10 Perpendicular from a point to a line (3D) (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 0 \\ -1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$.

The point P has coordinates $(-2, -2, 2)$.
Find the coordinates of the foot F of the perpendicular from P to l, and the exact distance PF.

F = (a + td) for some t, so $\mathrm{FP} = \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix} - t\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$
$\mathrm{FP} \cdot \mathbf{d} = 0$: $-4 - t(4) = 0$, so $t = -1$
F = $(-2, -1, 5)$
$\mathrm{PF} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$, so $\mathrm{PF} = \sqrt{10}$
**Answer: F = $(-2, -1, 5)$, PF = $\sqrt{10}$**