Day 335 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has its centre in the first quadrant and touches both the x-axis and the y-axis. It passes through the point (4, 8). Find the two possible equations of the circle.

Centre (r, r), radius r: $(x - r)^2 + (y - r)^2 = r^2$.

Substitute (4, 8): $(4 - r)^2 + (8 - r)^2 = r^2$.

$r^2 - 24r + 80 = 0$.

$(r - 4)(r - 20) = 0$, so r = 4 or r = 20.

Answer: $(x - 4)^2 + (y - 4)^2 = 16$ or $(x - 20)^2 + (y - 20)^2 = 400$

2 One-tailed binomial test with data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A seed supplier says that 50% of its seeds germinate. 25 seeds are planted, and 10 germinate.
Test, at the 5% significance level, the claim that the proportion is lower.
Give your conclusion in context.

H_0: p = 0.5, H_1: p < 0.5. Let X ~ B(25, 0.5)

$P(X \leq 10) = 0.212$

0.212 is greater than 5%, so do not reject H_0

There is insufficient evidence that the proportion is lower than the claimed 0.5

Answer: Do not reject H_0: p-value 0.212 > 5%

3 Integrals of the form f'(x)/f(x) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$$\int_0^2 \frac{10x - 10}{x^2 - 2x + 3}\,dx$$

Numerator = 5 × derivative of denominator

Integral = $[5 \ln(x^2 - 2x + 3)]$

$= 5(\ln 3 - \ln 3)$

Answer: 0

4 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|x + 1| < 3x + 5$.

Boundary values: $x + 1 = 3x + 5$ gives $x = -2$;

$x + 1 = -(3x + 5)$ gives $x = -\frac{3}{2}$.

Also the corner of $|x + 1|$ is at $x = -1$.

Testing each region (check the graph of each side) gives the solution set.

Answer: $x > -\frac{3}{2}$

5 Rough slope: up then back down (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 7 m s^{-1} up a line of greatest slope of a rough plane inclined at α to the horizontal, where $\sin\alpha = \frac{3}{5}$ and $\mu = 0.4$.

Find (a) the distance it travels up the plane, to 3 s.f., (b) its speed on returning to its starting point. Take g = 9.8 m s^{-2}.

$\cos\alpha = \frac{4}{5}$. Up the plane: $R = mg\cos\alpha$, so $a_{up} = g(\sin\alpha + \mu\cos\alpha) = 9.8(\frac{3}{5} + 0.4 \times \frac{4}{5}) = 9.016$

$0 = 7^2 - 2 \times 9.016 \times d$, so $d = \frac{49}{18.032} = 2.717$

Down the plane: $a_{down} = g(\sin\alpha - \mu\cos\alpha) = 9.8(\frac{3}{5} - 0.4 \times \frac{4}{5}) = 2.744$

$v^2 = 2 \times 2.744 \times 2.7174 = 14.91$, so v = 3.86

Answer: (a) 2.72 m (b) 3.86 m s^{-1}

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6 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both
$x^2 + x - 12 \geq 0$ and $3x - 2 > 4$.

Quadratic: $(x + 4)(x - 3) \geq 0$ gives $x \leq -4$ or $x \geq 3$.
Linear: $3x - 2 > 4$ gives $x > 2$.
Both must hold: take the overlap of the two sets.
Answer: $x \geq 3$

7 Partial fractions then integrate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Show that $\frac{x^2 + 4x - 1}{(x + 5)(x + 4)} \equiv 1 + \frac{A}{x + 5} + \frac{B}{x + 4}$.

(b) Hence find the exact value of $\int_0^3 \frac{x^2 + 4x - 1}{(x + 5)(x + 4)}\,dx$.

Numerator = $(x + 5)(x + 4) + A(x + 4) + B(x + 5)$
$x = -5$ gives $A = -4$; $x = -4$ gives $B = -1$
Integral = $[x + -4\ln(x + 5) - 1\ln(x + 4)]$
Substitute limits 0 and 3
Answer: A = −4, B = −1; $3 - 10\ln 2 + 4\ln 5 - \ln 7$

8 Most likely value of a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(18, \frac{2}{5})$.

(a) Show that $P(X = r + 1) \div P(X = r) = \frac{18 - r}{r + 1} \times \frac{2}{3}$.

(b) Find the most likely value of X and its probability (3 s.f.).

$P(X = r + 1) \div P(X = r) = \frac{C(18, r + 1)}{C(18, r)} \times \frac{p}{q} = \frac{18 - r}{r + 1} \times \frac{p}{q}$

With $p = \frac{2}{5}$ and $q = \frac{3}{5}$, $p \div q = \frac{2}{3}$

The ratio exceeds 1 when $(18 - r)\frac{2}{5} > (r + 1)\frac{3}{5}$, that is $r < 6.6$

So probabilities rise up to r = 7 and then fall; the mode is 7
P(X = 7) = 0.189
Answer: 7 with probability 0.189

9 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 + 2xy + 3y^2 = 96$.
Find the coordinates of the points on C where the tangent is parallel to the y-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + 2y}{2x + 6y}$

Vertical tangent when $2x + 6y = 0$, so $x = -\frac{6y}{2}$

Substitute into the equation of C: $x^2 + (2xy) + (3y^2) = 96$
Solve for the remaining coordinate
Answer: (−12, 4) and (12, −4)

10 Particle on a smooth slope joined to hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (1 kg) and B (2 kg) are joined by a light string over a smooth pulley (see diagram). A lies on a smooth slope, $\sin\alpha = \frac{4}{5}$, and B is 4 m above the ground.

Released from rest, find the further distance A moves up the slope after B lands (A does not reach the pulley).

A: $T - 1 \times 9.8 \times \frac{4}{5} = 1\,a$. B: $2 \times 9.8 - T = 2\,a$

Adding: $9.8(2 - 1 \times \frac{4}{5}) = 3\,a$, so $a = 3.92$

$T = 2(9.8 - 3.92) = 11.76$

When B lands: $v^2 = 2 \times 3.92 \times 4 = 31.36$

Then A decelerates at $g\sin\alpha = 7.84$: further distance $= \frac{31.36}{2 \times 7.84} = 2$

Answer: 2 m