Day 331 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Range and domain by completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$g(x) = \sqrt{-x^2 + 7x - 10}$.
Find the largest possible domain of g, and the corresponding range.

Need $-x^2 + 7x - 10 \geq 0$, i.e. $(x - 2)(x - 5) \leq 0$.
Domain: $2 \leq x \leq 5$.

Complete the square: $-x^2 + 7x - 10 = \frac{9}{4} - (x - \frac{7}{2})^2$.

Greatest value of $g = \sqrt{\frac{9}{4}} = \frac{3}{2}$, and least value is 0.

Answer: $2 \leq x \leq 5$; $0 \leq g(x) \leq \frac{3}{2}$

2 Related rates of change (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The volume of a sphere increases at a constant rate of 128π cm^3s^{-1}.
When the radius is 4 cm, find the rate of increase of (a) the radius,
(b) the surface area. Volume = $\frac{4}{3}\pi r^3$.

$dV/dt = 4\pi r^2\, dr/dt$, so $128\pi = 4\pi \times 16 \times dr/dt$
$dr/dt = 2$
$A = 4\pi r^2$, so $dA/dt = 8\pi r\, dr/dt = 8\pi \times 4 \times 2$
Answer: (a) 2 cm s^{-1} (b) 64π cm^2s^{-1}

3 Projectile on level ground (AQA, Edexcel, OCR, MEI, Eduqas)

A shell is projected from a point O on horizontal ground with speed 49 m s^{-1} at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Find (a) the greatest height, (b) the speed when it is at half of this height (3 s.f.). Take g = 9.8 m s^{-2}.

Vertical: $0 = 39.2^2 - 2 \times 9.8 \times H$, so $H = 78.4$
At height $\frac{H}{2} = 39.2$: $v_y^2 = 39.2^2 - 2 \times 9.8 \times 39.2 = 768.32$
$v_x = 29.4$ (constant), so $speed^2 = 864.36 + 768.32 = 1632.68$
Answer: (a) 78.4 m (b) 40.4 m s^{-1}

4 One-tailed binomial test with data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A manufacturer says that 25% of its screens have no flaws. A sample of 40 screens is checked, and 12 have no flaws.
Test, at the 5% significance level, the claim that the proportion is higher.
Give your conclusion in context.

H_0: $p = 0.25$, H_1: $p > 0.25$. Let $X \sim B(40, 0.25)$
$P(X \geq 12) = 1 - P(X \leq 11) = 0.285$
0.285 is greater than 5%, so do not reject H_0
There is insufficient evidence that the proportion is higher than the claimed 0.25
Answer: Do not reject H_0: p-value 0.285 > 5%

5 Equations in disguise (substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 5\sqrt{x} + 4 = 0$.

Let $u = \sqrt{x}$: $u^2 - 5u + 4 = 0$.
$(u - 4)(u - 1) = 0$, so $\sqrt{x} = 4$ or $\sqrt{x} = 1$.
Square: $x = 16$ or $x = 1$.
Answer: $x = 1$ or $x = 16$

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6 Grouped median with an unknown frequency (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the times, t minutes, taken by some runners.
Using linear interpolation, the median is estimated as 26.5 minutes. Find the value of k.

Total = 31 + k, so the median class is 20 to 30

Median = $20 + \frac{\frac{(31+k)}{2} - (6+k)}{10} \times 10 = 26.5$

Simplify: $9.5 - k/2 = 6.5$

$k = 6$

Answer: k = 6

7 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 3x^3\sqrt{252 - x}$, for $0 < x < 252$, has one stationary point. Find its exact coordinates and determine its nature.

$dy/dx = \frac{3x^2(1512 - 7x)}{2\sqrt{252 - x}}$

$dy/dx = 0$: $1512 - 7x = 0$, so $x = 216$

$y = 3 \times 216^3 \times 6$

Gradient changes from positive to negative: a maximum

Answer: (216, 181398528), maximum

8 Recurrence: limit and convergence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$u_{n+1} = \frac{2}{3}u_n + 4$, $u_1 = 16$.

(a) Find the limit L of the sequence.
(b) Find the least n such that $|u_n - L| < 0.01$.

(a) At the limit $L = \frac{2}{3}L + 4$, so $L(1 - \frac{2}{3}) = 4$.

$L = 12$.

(b) $u_n - L = (u_1 - L)(\frac{2}{3})^{n-1} = 4(\frac{2}{3})^{n-1}$.

Need $4 \times (\frac{2}{3})^{n-1} < 0.01$, so $n - 1 > 14.777$.

The least n is 16.

Answer: (a) L = 12 (b) n = 16

9 Constant speed up a slope, force at an angle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of weight 75 N is pulled at constant speed up a line of greatest slope of a rough plane ($\tan\alpha = \frac{7}{24}$, $\mu = \frac{3}{4}$) by a force P N at angle β above the plane.

(a) Find P when $\tan\beta = \frac{4}{3}$. (b) Find the least possible value of P as β varies.

Perpendicular to the plane: $R + P\sin\beta = 75\cos\alpha$, so $R = 75 \times \frac{24}{25} - P\sin\beta$

Along the plane: $P\cos\beta = 75\sin\alpha + \mu R$, so $P(\cos\beta + \mu\sin\beta) = 75(\sin\alpha + \mu\cos\alpha)$

(a) $P(\frac{3}{5} + \frac{3}{4} \times \frac{4}{5}) = 75(\frac{7}{25} + \frac{3}{4} \times \frac{24}{25})$, so $P = 62.5$

(b) P is least when $\cos\beta + \mu\sin\beta$ is greatest, i.e. $\sqrt{(1 + \mu^2)} = \frac{5}{4}$ (when $\tan\beta = \frac{3}{4}$)

$P_{min} = 75(\frac{7}{25} + \frac{3}{4} \times \frac{24}{25}) \div \frac{5}{4}$

Answer: (a) 62.5 N (b) 60 N

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10 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^1 x^2 e^{-x}\,dx$

Parts twice: $u = x^2$, $dv/dx = e^{-x}$

Integral = $[\frac{x^2}{-1}e^{-x}] - \int \frac{2x}{-1}e^{-x}\,dx$

Repeat with $u = x$ on the remaining integral

Answer: $2 - 5e^{-1}$