**Day 327** **A Level Maths: S Tier 10-a-day: Model Solutions**

## 1 Waiting for the r-th success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each shot Amira takes scores with probability $\frac{2}{5}$, independently.

(a) Find the probability that her second score comes on shot number 9.
(b) Find the probability that her second score comes on or before shot number 9.

(a) second score on shot 9: exactly 1 scores in the first 8 shots, then a score

$C(8, 1)(\frac{2}{5})^1(\frac{3}{5})^7 \times \frac{2}{5}$

$= 0.0358$

(b) On or before shot 9 means at least 2 scores in 9 shots: $X \sim B(9, \frac{2}{5})$

$P(X \geq 2) = 1 - P(X \leq 1) = 0.929$

**Answer: (a) 0.0358 (b) 0.929**

## 2 Compound interest and savings (GP) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is dropped from a height of 4 m. After each bounce it rises to $\frac{3}{5}$ of its previous height.

(a) Find the total distance it travels before it stops.
(b) After how many bounces does it rise less than 0.01 m?

(a) Down H, then up and down each time: $H + 2Hr + 2Hr^2 + ...$

$= H + \frac{2Hr}{1 - r} = 4 + \frac{2 \times 4 \times \frac{3}{5}}{1 - \frac{3}{5}} = 16.$

(b) Rebound height after n bounces $= 4(\frac{3}{5})^n < 0.01.$

$n > \frac{\log(0.0025)}{\log(0.6)} = 11.73$, so $n = 12$.

**Answer: (a) 16 m (b) 12 bounces**

## 3 Velocity-time graph with a return (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the velocity-time graph of a particle P moving on a straight line (velocity in m $s^{-1}$, time in s). At time 16 s, P is back at its starting point.
Find (a) the value of W, (b) the greatest distance of P from its starting point.

Net area = 0 (back at start): $\frac{1}{2} \times 5 \times 12 + \frac{1}{2}(12 - W)(4) - W(7) = 0$
$30 + 24 - 2W - 7W = 0$, so $9W = 54$
$W = 6$

v = 0 where the line from (5, 12) to (9, −6) crosses the axis: $t = 5 + 4 \times \frac{12}{18} = \frac{23}{3}$

Greatest distance = area of triangle $= \frac{1}{2} \times \frac{23}{3} \times 12$

**Answer: (a) W = 6 (b) 46 m**

## 4 Differentiating a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 5^x - 3x$ has one stationary point.
Find its coordinates to 3 s.f. and show that it is a minimum.

$dy/dx = 5^x \ln 5 - 3 = 0$

$5^x = \frac{3}{\ln 5}$, $x = \frac{1}{\ln 5} \ln(\frac{3}{\ln 5}) = 0.387$

$d^2y/dx^2 = 5^x(\ln 5)^2 > 0$ for all x: minimum

$y = \frac{3}{\ln 5} - 3 \times x = 0.703$

**Answer: (0.387, 0.703)**

## 5 Constant speed up a slope, force at an angle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of weight 110 N is pulled at constant speed up a line of greatest slope of a rough plane ($\tan\alpha = \frac{12}{5}$, $\mu = \frac{5}{12}$) by a force P N at angle $\beta$ above the plane.

(a) Find P when $\tan\beta = \frac{5}{12}$. (b) Find the least possible value of P as $\beta$ varies.

Perpendicular to the plane: $R + P\sin\beta = 110\cos\alpha$, so $R = 110 \times \frac{5}{13} - P\sin\beta$

Along the plane: $P\cos\beta = 110\sin\alpha + \mu R$, so $P(\cos\beta + \mu\sin\beta) = 110(\sin\alpha + \mu\cos\alpha)$

(a) $P(\frac{12}{13} + \frac{5}{12} \times \frac{5}{13}) = 110(\frac{12}{13} + \frac{5}{12} \times \frac{5}{13})$, so $P = 110$

(b) P is least when $\cos\beta + \mu\sin\beta$ is greatest, i.e. $\sqrt{(1+\mu^2)} = \frac{13}{12}$ (when $\tan\beta = \frac{5}{12}$)

$P_{min} = 110(\frac{12}{13} + \frac{5}{12} \times \frac{5}{13}) \div \frac{13}{12}$

**Answer: (a) 110 N (b) 110 N**

## 6 Quadratic sequence from recurrence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$u_{n+1} = u_n + 3n - 5$, $u_1 = 6$.

(a) Show that $u_n = an^2 + bn + c$ and find a, b, c.

(b) Find the least n for which $u_n > 689$.

$u_n = u_1 + \sum_{k=1}^{n-1}(3k - 5) = 6 + 3 \times \frac{(n-1)n}{2} - 5(n-1)$.

$= \frac{3}{2}n^2 - \frac{13}{2}n + 11$.

So $a = \frac{3}{2}$, $b = -\frac{13}{2}$, $c = 11$.

$u_{23} = 655 \leq 689$ and $u_{24} = 719 > 689$.

**Answer: (a) $a = \frac{3}{2}$, $b = -\frac{13}{2}$, $c = 11$ (b) $n = 24$**

## 7 Triangle in 3D: angle and area (CIE (9709); Further Maths on other boards)

The points A, B, C have coordinates (0, −1, 2), (5, 0, −3), (5, 1, −1).

Find angle ABC to 1 d.p., and the exact area of triangle ABC.

$BA = \begin{pmatrix}-5\\-1\\5\end{pmatrix}$, $BC = \begin{pmatrix}0\\1\\2\end{pmatrix}$; $BA \cdot BC = 9$

$|BA| = \sqrt{51}$, $|BC| = \sqrt{5}$

$\cos ABC = \frac{9}{\sqrt{51}\sqrt{5}} = 0.5636$, so $ABC = 55.7°$

$|BA \times BC| = |\begin{pmatrix}-7\\10\\-5\end{pmatrix}| = \sqrt{174}$

Area $= \frac{1}{2}|BA \times BC|$

**Answer: ABC = 55.7°, area $= \frac{1}{2}\sqrt{174}$**

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## 8 Distribution of the larger of two scores (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two fair 4-sided dice, numbered 1 to 4, are rolled. M is the larger of the two scores.

(a) Show that $P(M = m) = \frac{(2m - 1)}{16}$ for m = 1, 2, ..., 4.

(b) Find P(M = 3) and E(M).

(a) M ≤ m means both scores ≤ m: $P(M \leq m) = (m/4)^2$, so $P(M = m) = \frac{m^2 - (m - 1)^2}{16} = \frac{(2m - 1)}{16}$

(b) $P(M = 3) = \frac{5}{16}$

$E(M) = \Sigma\, m\, P(M = m) = \frac{1}{16} \Sigma\, m(2m - 1)$

$= \frac{25}{8}$

**Answer: (b) $P(M = 3) = \frac{5}{16}$, $E(M) = \frac{25}{8}$**

## 9 Conditions on the roots of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The roots of $x^2 + kx + 16 = 0$ are in the ratio 4 : 1. Find the possible values of k.

Let the roots be α and 4α.

Sum: $5\alpha = -k$. Product: $4\alpha^2 = 16$, so $\alpha^2 = 4$.

$\alpha = \pm 2$, so $k = -5\alpha = \mp 10$.

**Answer: k = ±10**

## 10 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population N satisfies $\frac{dN}{dt} = -0.005N^2$, with N = 100 when t = 0.

Find N when t = 2, and the time taken for N to halve.

$\int N^{-2}\, dN = -0.005 \int dt$ gives $-\frac{1}{N} = -0.005t + c$

t = 0: $c = -\frac{1}{100}$, so $\frac{1}{N} = 0.005t + \frac{1}{100}$

t = 2: $\frac{1}{N} = \frac{1}{100} + \frac{1}{100}$; halving: $0.005t = \frac{1}{100}$

**Answer: N = 50; t = 2**