Day 326 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Perpendicular lines and area (diagram) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the diagram, line l_1 passes through A and B.
Line l_2 is perpendicular to l_1 at B and meets the
x-axis at C. Find the coordinates of C and the area of triangle ABC.

Gradient of $l_1 = \frac{3}{1-4} = -1$.
Gradient of $l_2 = 1$. Equation: $y - 3 = 1(x - 1)$.
$y = 0$: $x = -2$, so C$(-2, 0)$.
AB $= 3\sqrt{2}$, BC $= 3\sqrt{2}$: area = ½ × AB × BC.
Answer: C$(-2, 0)$; area 9

2 Resultant of forces: unknown constant and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-3\mathbf{i} - 7\mathbf{j})$ N, $(3\mathbf{i} + k\mathbf{j})$ N and $(-\mathbf{i} - 8\mathbf{j})$ N act on a particle of mass 4 kg. The resultant is parallel to $(-\mathbf{i} - 15\mathbf{j})$.
Find (a) the value of k, (b) the magnitude of the acceleration of the particle, to 3 s.f.

Resultant $= (-1)\mathbf{i} + (-15 + k)\mathbf{j}$
Parallel to $(-\mathbf{i} - 15\mathbf{j})$: $\frac{-15+k}{-1} = \frac{-15}{-1}$, so $k = 0$
$\mathbf{R} = (-\mathbf{i} - 15\mathbf{j})$ N, so $|R| = \sqrt{(-1^2 + -15^2)} = 15.03$
$a = |R| \div 4$
Answer: (a) $k = 0$ (b) 3.76 m s^{-2}

3 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\tan(2\theta + 15°) = -\frac{1}{\sqrt{3}}$
for $0° \leq \theta \leq 180°$.

Let $u = 2\theta + 15°$; then $\tan u = -\frac{1}{\sqrt{3}}$
Base solutions: $u = -30°$ (+ multiples of 180°)
$\theta = (u - 15°) \div 2$, keeping those in range
Answer: θ = 67.5°, 157.5°

4 Sums of multiples (arithmetic series) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the integers from 1 to 233
inclusive that are divisible by 6 or by 8, but not by both.

Multiples of 6: 38 terms (6 to 228): sum = 4446.
Multiples of 8: 29 terms: sum = 3480.
Multiples of both, i.e. of 24: 9 terms: sum = 1080.
Required sum = 4446 + 3480 − 2 × 1080.
Answer: 5766

5 Poisson approximation to a binomial (CIE)

X ~ B(150, 0.02).
(a) Explain why a Poisson approximation is suitable and state its distribution.
(b) Use it to estimate P(X > 2), to 4 s.f.
(c) Find the exact value of P(X > 2), to 4 s.f.

n = 150 is large and p = 0.02 is small, with np = 3
X is approximately Po(3)
$P(X > 2) \approx 1 - P(Y \leq 2) = 0.5768$
Exact: $1 - P(X \leq 2)$ with X ~ B(150, 0.02) = 0.5791
Answer: (a) Po(3) (b) 0.5768 (c) 0.5791

6 Constant acceleration in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves with constant acceleration. At t = 0 seconds P is at $(\mathbf{i} + 2\mathbf{j})$ m with velocity $(4\mathbf{i} - 12\mathbf{j})$ m s^{-1}. At t = 8, P has velocity $(4\mathbf{i} + 12\mathbf{j})$ m s^{-1}.
Find (a) the acceleration of P, (b) the time at which P moves parallel to $(\mathbf{i})$, (c) the position vector of P at that time.

$a = \frac{v - u}{t} = \frac{(4\mathbf{i} + 12\mathbf{j}) - (4\mathbf{i} - 12\mathbf{j})}{8} = (3\mathbf{j})$

$v = (4\mathbf{i} - 12\mathbf{j}) + t(3\mathbf{j})$

Parallel to $(\mathbf{i})$: $(4) \times 0 = (3t - 12) \times 1$, giving t = 4

$r = (\mathbf{i} + 2\mathbf{j}) + 4(4\mathbf{i} - 12\mathbf{j}) + \frac{1}{2} \times 4^2 (3\mathbf{j}) = (17\mathbf{i} - 22\mathbf{j})$

Answer: (a) $(3\mathbf{j})$ m s^{-2} (b) t = 4 (c) $(17\mathbf{i} - 22\mathbf{j})$ m

7 Transformations: new equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 - 4x - 2$ is translated by $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$
and then a reflection in the y-axis.
Find the equation of the resulting curve in the form $y = ax^2 + bx + c$.

Translation by $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$: $y = f(x + 3) - 2 = x^2 + 2x - 7$.

Then a reflection in the y-axis: $x \to -x$.
Expand and simplify to give the quadratic.

Answer: $y = x^2 - 2x - 7$

8 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^{\pi} x \sin 2x \, dx$

Let u = x and dv/dx = sin 2x

Integral = [uv] $\mp$ ... using $v = -\frac{1}{2}\cos 2x$

Integral = $[-\frac{x \cos 2x}{2} + \frac{\sin 2x}{4}]$

Answer: $-\frac{1}{2}\pi$

9 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The journey time is N(μ, 64) with known variance 64. The mean was claimed to be 64 minutes.
A random sample of 49 has mean 62. Test at the 5% level whether the mean is greater than 64.

H_0: μ = 64, H_1: μ > 64. Under H_0, the sample mean ~ N(64, $\frac{64}{49}$)

$z = \frac{62 - 64}{8 \div \sqrt{49}} = -1.750$

p-value = P(Z > −1.750) = 0.960

0.960 > 5%, so do not reject H_0

There is insufficient evidence at the 5% level that the mean has changed as claimed

Answer: Do not reject H_0 (z = −1.750, p-value 0.960)

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10 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \frac{1}{18 + 15\sin\theta + 8\cos\theta}$

(a) Write $15\sin\theta + 8\cos\theta$ as $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.

(b) Find the greatest and least values of $f(\theta)$, and the smallest positive θ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{15^2 + 8^2} = 17$, $\tan\alpha = \frac{8}{15}$, $\alpha = 28.1°$

Denominator $= 18 + 17\sin(\theta + 28.1°)$, which lies between 1 and 35

Greatest $f(\theta) = \frac{1}{1} = 1$, when $\sin(\theta + \alpha) = -1$

Least $f(\theta) = \frac{1}{35} = \frac{1}{35}$, when $\sin(\theta + \alpha) = 1$

$\sin(\theta + \alpha) = -1$: $\theta + 28.1° = 270°$, so $\theta = 241.9°$

Answer: (a) $17\sin(\theta + 28.1°)$ (b) greatest 1, least $\frac{1}{35}$, $\theta = 241.9°$