Day 325 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Screening test and Bayes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A disease is present in 2% of a population. A test is positive for 90% of those with it and for 5% of those without it.
Find P(positive) and P(has it | positive).

P(positive) = 0.02 × 0.9 + 0.98 × 0.05
= 0.018 + 0.049 = 0.067

P(has it | positive) = $\frac{0.018}{0.067}$

= 0.2687

Answer: (a) 0.067 (b) 0.269

2 Three-stage journey (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train accelerates uniformly from rest at 2.4 m s^{-2} to a speed of V m s^{-1}, travels at this speed, then decelerates uniformly at 1.5 m s^{-2} to rest.
The journey takes 50 s and covers 888 m. Find V and the time spent at constant speed.

Time accelerating = $\frac{V}{2.4}$, time decelerating = $\frac{V}{1.5}$, so time at constant speed = $50 - \frac{V}{2.4} - \frac{V}{1.5}$

Area under the speed-time graph: $888 = 50V - \frac{V^2}{4.8} - \frac{V^2}{3}$

Multiply by 2ab: $3.9V^2 - 360V + 6393.6 = 0$

V = 24 or V = $\frac{888}{13}$

V = $\frac{888}{13}$ gives time at constant speed −24 s, which is negative, so V = 24

Time at constant speed = 50 − 10 − 16 = 24 s

Answer: V = 24, 24 s

3 Logarithmic equations with ln (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve 2 ln x = ln(x + 6) + ln 8,
stating clearly why one root is rejected.

ln x^2 = ln(8(x + 6)), so $x^2 = 8x + 48$

$x^2 - 8x - 48 = 0$, $(x - 12)(x + 4) = 0$

x = −4 is rejected since ln x is undefined for x < 0

Answer: x = 12

4 Binomial expansion (integer powers) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(2x - 2)(1 - 2x)^6$.

$(1 - 2x)^6$: x^2 term has coefficient C(6,2) × $(-2)^2$ = 60, x^1 term has coefficient C(6,1) × $(-2)^1$ = −12.

x^2 in the product: (−2) × 60 + 2 × (−12).

= −144.

Answer: −144

5 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2$, $y = 2t^3 + 18t$.
Find the coordinates of the points on the curve where
the gradient is −6.

dx/dt = 4t, dy/dt = $6t^2 + 18$, so dy/dx = $\frac{6t^2 + 18}{4t}$

Set equal to −6: $6t^2 + 24t + 18 = 0$

t = −3 or t = −1

Answer: (18, −108) and (2, −20)

6 Combined mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Class R has 12 students with mean mark 72 and standard deviation 4. Class S has 15 students with mean mark 80 and standard deviation 4.
(a) Find the mean mark of all 27 students.
(b) Find the standard deviation of all 27 students.

Σx for R = $12 \times 72 = 864$, Σx for S = $15 \times 80 = 1200$

Combined mean = $\frac{2064}{27} = 76.4$

$\Sigma x^2 = n(\sigma^2 + \text{mean}^2)$: $12(16 + 5184) + 15(16 + 6400) = 158640$

Variance = $\frac{158640}{27} - 76.444^2 = 31.802$, so $\sigma = 5.639$

Answer: (a) 76.4 (b) 5.64

7 Variable force in vector form (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P of mass 5 kg moves under a single force F N, where $\mathbf{F} = (10t - 10)\mathbf{i} + (30t^2 - 30)\mathbf{j}$ and t is the time in seconds. At t = 0, P is at the origin with velocity $(4\mathbf{i} + 2\mathbf{j})$ m s^{-1}.
Find (a) the velocity of P at t = 3, (b) the position vector of P at t = 3.

$\mathbf{a} = \frac{\mathbf{F}}{5} = (2t - 2)\mathbf{i} + (6t^2 - 6)\mathbf{j}$

$\mathbf{v} = \int \mathbf{a}\, dt = (t^2 - 2t + 4)\mathbf{i} + (2t^3 - 6t + 2)\mathbf{j}$ (using $\mathbf{v}(0) = (4\mathbf{i} + 2\mathbf{j})$)
At t = 3: $\mathbf{v} = (7\mathbf{i} + 38\mathbf{j})$

$\mathbf{r} = \int \mathbf{v}\, dt = (\frac{1}{3}t^3 - t^2 + 4t)\mathbf{i} + (0.5t^4 - 3t^2 + 2t)\mathbf{j}$

At t = 3: $\mathbf{r} = (12\mathbf{i} + 19.5\mathbf{j})$

Answer: (a) $(7\mathbf{i} + 38\mathbf{j})$ m s^{-1} (b) $(12\mathbf{i} + 19.5\mathbf{j})$ m

8 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|-x + 2| \leq |-3x - 6|$.

Square both sides: $(-x + 2)^2 \leq (-3x - 6)^2$.
$(2x + 8)(-4x - 4) \leq 0$.
Critical values $x = -4$ and $x = -1$.
The quadratic has negative coefficient of x^2, so choose the outer region.
Answer: $x \leq -4$ or $x \geq -1$

9 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 - 3xy + 5y^2 = 99$.
Find the coordinates of the points on C where the tangent is parallel to the x-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + -3y}{-3x + 10y}$

Horizontal tangent when $2x + -3y = 0$, so $y = \frac{2x}{3}$

Substitute into the equation of C: $x^2 + (-3xy) + (5y^2) = 99$
Solve for the remaining coordinate
Answer: $(-9, -6)$ and $(9, 6)$

10 Conditions on the roots of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The roots of $x^2 + kx + 21 = 0$ differ by 4.
Find the possible values of k.

Let the roots be α and $\alpha + 4$.

Product: $\alpha(\alpha + 4) = 21$, so $\alpha^2 + 4\alpha - 21 = 0$.
$(\alpha - 3)(\alpha + 7) = 0$, so $\alpha = 3$ or $\alpha = -7$.
Sum: $2\alpha + 4 = -k$, so $k = -(2\alpha + 4) = -10$ or 10.
Answer: $k = \pm 10$