Day 321 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Force applied then removed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 5 kg is at rest on rough horizontal ground ($\mu = 0.1$). A horizontal force of 8.4 N acts on it for 7 s and is then removed.

Find (a) the speed when the force is removed, (b) the total distance travelled before the box stops. Take $g = 9.8$ m s^{-2}.

Friction = $\mu mg = 0.1 \times 5 \times 9.8 = 4.9$ N

While pushed: $8.4 - 4.9 = 5a$, so $a = 0.7$

$v = 0.7 \times 7 = 4.9$; $s_1 = \frac{1}{2} \times 0.7 \times 7^2 = 17.15$

After release: deceleration = $\mu g = 0.98$, so $s_2 = \frac{4.9^2}{2 \times 0.98} = 12.25$

Total distance = 17.15 + 12.25

Answer: (a) 4.9 m s^{-1} (b) 29.4 m

2 Line tangent to a hyperbola (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = \frac{x + 11}{x + 3}$.

(a) Write down the equations of the asymptotes of C.

(b) The line $y = -2x + k$ is a tangent to C.

Find the possible values of k.

(a) Vertical: $x = -3$. As $x \to \infty$, $y \to 1$, so $y = 1$.

(b) $x + 11 = (-2x + k)(x + 3)$.

$-2x^2 + (k - 7)x + (3k - 11) = 0$.

Tangent: $b^2 - 4ac = 0$, which simplifies to $k^2 + 10k - 39 = 0$.

Answer: (a) $x = -3$, $y = 1$ (b) $k = -13$ or $k = 3$

3 Stone thrown from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 9 m s^{-1} from the top of a cliff. The top of the cliff is 17.1 m above the sea.

Find (a) the time taken for the stone to reach the sea, (b) the total distance travelled by the stone, to 3 s.f. Take $g = 9.8$ m s^{-2}.

Take upwards as positive: $-17.1 = 9t - 4.9t^2$, so $4.9t^2 - 9t - 17.1 = 0$

$t = \frac{9 + \sqrt{(9^2 + 19.6 \times 17.1)}}{9.8} = \frac{9 + 20.4}{9.8} = 3$

Height risen above the cliff top = $\frac{9^2}{(2 \times 9.8)} = 4.13$ m

Total distance = $2 \times 4.13 + 17.1$

Answer: (a) 3 s (b) 25.4 m

4 Self-inverse function (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{kx - 1}{x - 2}$ for $x \neq 2$. Given that f is self-inverse, find k, and hence solve $f(x) = x$.

Let $y = f(x)$: $y(x - 2) = kx - 1$, so $x(y - k) = 2y - 1$.

$f^{-1}(x) = \frac{2x - 1}{x - k}$.

$f = f^{-1}$ requires $k = 2$.

$f(x) = x$: $2x - 1 = x(x - 2)$, so $x^2 - 4x + 1 = 0$.

$x = 2 \pm \sqrt{3} = 2 \pm \sqrt{3}$.

Answer: $k = 2$; $x = 2 \pm \sqrt{3}$

5 Tangent and axes triangle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tangent to $y = \frac{x^2 + 4}{x + 4}$ at the point where x = 1
meets the axes at A and B. Find the exact area of triangle OAB,
where O is the origin.

At x = 1: y = 1. Differentiate: gradient = $\frac{1}{5}$

Tangent: $y - 1 = \frac{1}{5}(x - 1)$, so $y = \frac{1}{5}x + \frac{4}{5}$

y-intercept $\frac{4}{5}$; x-intercept -4

Area = $\frac{1}{2} \times |\frac{4}{5}| \times |-4|$

Answer: $\frac{8}{5}$

6 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The daily output of a machine is N(μ, 4) with known variance 4. The mean was claimed to be 56 units.
A random sample of 36 has mean 55.75. Test at the 10% level whether the mean is different from 56.

H_0: μ = 56, H_1: μ ≠ 56. Under H_0, the sample mean ~ N(56, $\frac{4}{36}$)

$z = \frac{55.75 - 56}{2 \div \sqrt{36}} = -0.750$

One tail: P(Z < −0.750) = 0.227, compared with 5%

0.227 > 5%, so do not reject H_0

There is insufficient evidence at the 10% level that the mean has changed as claimed

Answer: Do not reject H_0 (z = −0.750, p-value 0.227)

7 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_0^{\pi} x \cos x \, dx$

Let u = x and dv/dx = cos x

Integral = [uv] ∓ ... using v = sin x

Integral = $[\frac{x \sin 1x}{1} + \frac{\cos 1x}{1}]$

Answer: −2

8 Exponential and power models by logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data satisfy $y = ab^x$. The line of best fit for lg y against x is lg y = 0.9 + 0.85x.
(a) Find a and b to 3 significant figures.
(b) Estimate y when x = 16, to 3 significant figures.
(c) Interpret the value of b.

$y = ab^x$ gives lg y = lg a + x lg b

lg a = 0.9 so $a = 10^{0.9} = 7.943$

lg b = 0.85 so $b = 10^{0.85} = 7.079$

x = 16: lg y = 0.9 + 0.85 × 16 = 14.5

$y = 10^{14.5} = 316000000000000$

b is the factor y is multiplied by each time x increases by 1 (growth of 610%)

Answer: (a) a = 7.94, b = 7.08 (b) 316000000000000 (c) y grows by a factor of 7.08 per unit of x

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9 Parametric curve and line intersection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has parametric equations

$x = 2t$, $y = 3t^2$. The line $y = \frac{9}{2}x - 6$

meets C at A and B. Find the exact length AB.

Substitute: $3t^2 = 9t - 6$.

$3t^2 - 9t + 6 = 0 \Rightarrow t = 1$ or $t = 2$.

A(2, 3), B(4, 12).

$AB^2 = (-2)^2 + (-9)^2 = 85$.

Answer: AB = $\sqrt{85}$

10 Lines in 3D: intersection and angle (CIE (9709); Further Maths on other boards)

Line l_1: $\mathbf{r} = \begin{pmatrix} 8 \\ 0 \\ -1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}$,

line l_2: $\mathbf{r} = \begin{pmatrix} 9 \\ 2 \\ -4 \end{pmatrix} + t\begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$.

Show whether the lines meet, giving the point of intersection if they do, and find the acute angle between them to 1 d.p.

Equate components: $8 + 3s = 9 + 2t$; $3s = 2 + 2t$; $-1 - s = -4 - 2t$

The equations are inconsistent, so the lines do not meet (skew)

$\cos\varphi = \frac{|14|}{\sqrt{19}\sqrt{12}} = 0.9272$, $\varphi = 22.0°$

Answer: Lines do not meet (skew); angle 22.0°