Day 319 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The size of a rod is N(78, 10^2) mm. A rod is 'large' if it exceeds 90.
(a) Find the probability that a rod is large.
(b) 15 rods are chosen at random. Find the probability that at least 4 are large.

(a) P(X > 90) = P(Z > 1.200) = 0.1151
(b) Let Y = number large, Y ~ B(15, 0.1151)
P(Y ≥ 4) = 1 − P(Y ≤ 3) = 0.0848
Answer: (a) 0.115 (b) 0.0848

2 Lift, passenger and cable (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 500 kg, carrying a passenger of mass 70 kg, is supported by a vertical cable. The lift is moving upwards and accelerating at 0.4 m s^{-2}.
Find (a) the force the floor exerts on the passenger, (b) the tension in the cable. Take g = 9.8 m s^{-2}.

Take the direction of the acceleration as positive: it is upwards
Passenger: R − 70 × 9.8 = 70 × 0.4
R = 70 × (9.8 + 0.4) = 714
Lift and passenger: T − 570 × 9.8 = (500 + 70) × 0.4
T = 570 × 10.2 = 5814
Answer: (a) 714 N (b) 5814 N

3 Equations in disguise (substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x + 2\sqrt{x} - 3 = 0$.

Let $u = \sqrt{x}$: $u^2 + 2u - 3 = 0$.
$(u - 1)(u + 3) = 0$, so $\sqrt{x} = 1$ or $\sqrt{x} = -3$.
$\sqrt{x}$ cannot be negative, so reject $\sqrt{x} = -3$.
x = 1.
Answer: x = 1

4 Newton cooling model (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A drink cools according to $\theta = 20 + Ae^{-kt}$, where θ °C is the temperature after t minutes. Initially θ = 80, and after 5 minutes θ = 62.
Find A and k, then find when θ = 48 (3 s.f.).

t = 0: 20 + A = 80 so A = 60

t = 5: $20 + 60e^{-5k} = 62$ so $e^{-5k} = \frac{7}{10}$

$k = \frac{1}{5} \ln \frac{10}{7} = 0.0713$

$60\, e^{-kt} = 28$ so $t = \frac{1}{k} \ln \frac{15}{7}$

t = 10.7 minutes
Answer: A = 60, k = 0.0713, t = 10.7 minutes

5 Histogram: frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t seconds, of some runners.
Find (a) the total number of runners, (b) an estimate of the number with 13 ≤ t < 30.

Frequency = frequency density × width. Bar 26 to 34: height 7 squares, width 8, frequency 56
So one square of frequency density = 1 runners per second
Class frequencies = 72, 20, 4, 18, 56
Total = 170
Estimate = 1 × [5(3) + 1(4) + 3(6) + 7(4)] = 65
Answer: (a) 170 (b) 65

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6 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = 2x^3\sqrt{28 - x}$, for $0 < x < 28$, has one stationary point. Find its exact coordinates and determine its nature.

$\mathrm{d}y/\mathrm{d}x = \dfrac{2x^2(168 - 7x)}{2\sqrt{28 - x}}$

$\mathrm{d}y/\mathrm{d}x = 0$: $168 - 7x = 0$, so $x = 24$

$y = 2 \times 24^3 \times 2$

Gradient changes from positive to negative: a maximum

Answer: (24, 55296), maximum

7 Binomial approximation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(9 - x)^{-\frac{1}{2}}$ up to and including the term in x^2.

(b) Use $x = 0.01$ to estimate $\dfrac{1}{\sqrt{8.99}}$ to 5 decimal places.

(a) $(9 - x)^{-\frac{1}{2}} = \frac{1}{3}(1 - \frac{1}{9}x)^{-\frac{1}{2}}$.

$= \frac{1}{3} + \frac{1}{54}x + \frac{1}{648}x^2 + \ldots$

(b) $9 - x$ with $x = 0.01$ gives 8.99.

Substitute: $\frac{1}{3} + \frac{1}{54}(0.01) + \frac{1}{648}(0.01)^2$.

$= 0.33352$ (5 d.p.).

Answer: (a) $\frac{1}{3} + \frac{1}{54}x + \frac{1}{648}x^2$ (b) 0.33352

8 R-form: greatest and least values (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(\theta) = \dfrac{1}{32 + 24\sin\theta + 7\cos\theta}$

(a) Write $24\sin\theta + 7\cos\theta$ as $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.

(b) Find the greatest and least values of $f(\theta)$, and the smallest positive θ at which the greatest value occurs, to 1 d.p.

$R = \sqrt{24^2 + 7^2} = 25$, $\tan\alpha = \frac{7}{24}$, $\alpha = 16.3°$

Denominator $= 32 + 25\sin(\theta + 16.3°)$, which lies between 7 and 57

Greatest $f(\theta) = \frac{1}{7} = \frac{1}{7}$, when $\sin(\theta + \alpha) = -1$

Least $f(\theta) = \frac{1}{57} = \frac{1}{57}$, when $\sin(\theta + \alpha) = 1$

$\sin(\theta + \alpha) = -1$: $\theta + 16.3° = 270°$, so $\theta = 253.7°$

Answer: (a) $25\sin(\theta + 16.3°)$ (b) greatest $\frac{1}{7}$, least $\frac{1}{57}$, $\theta = 253.7°$

9 Car on a hill: power and maximum speed (CIE)

A car of mass 800 kg, with engine power 13.84 kW, climbs a straight road with $\sin\alpha = \frac{1}{20}$ against a constant resistance of 300 N.

Find (a) the greatest steady speed up the hill, (b) the acceleration at 10 m s^{-1}. Take $g = 9.8$ m s^{-2}.

Steady speed: driving force $= 300 + 800 \times 9.8 \times \frac{1}{20} = 692$ N

$P = F v$, so $v = \frac{13840}{692} = 20$

At 10 m s^{-1}: $F = \frac{13840}{10} = 1384$ N

$F - 692 = 800\,a$, so $a = \frac{692}{800}$

Answer: (a) 20 m s^{-1} (b) 0.865 m s^{-2}

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10 Remainder theorem (quadratic divisor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when x^5 is divided by
$(x - 1)(x - 2)$.

$x^5 = (x - 1)(x - 2)Q(x) + px + q$.
$x = 1$: $1 = p + q$.
$x = 2$: $32 = 2p + q$.
Subtract: $-p = -31$, so $p = 31$ and $q = -30$.
Answer: $31x - 30$