Day 316 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Outliers and the 1.5 × IQR rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A data set has lower quartile 15 and upper quartile 31. An outlier is a value more than 1.5 × IQR beyond the nearest quartile.
(a) Find the range of values that are not outliers.
(b) 15 observations have mean 16. The largest is 60, an outlier, and it is removed. Find the new mean.

IQR = 31 − 15 = 16, so 1.5 × IQR = 24
Lower limit = 15 − 24 = −9, upper limit = 31 + 24 = 55
$\Sigma x = 15 \times 16 = 240$, so without 60: $\Sigma x = 180$
New mean = $\frac{180}{14}$

Answer: (a) −9 to 55 (b) 12.86

2 Time above a given height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards from ground level. It is more than 171.5 m above the ground for 2 s.
Find the speed of projection and the greatest height reached. Take g = 9.8 m s^{-2}.

By symmetry, the ball takes 1 s to rise from height 171.5 m to its highest point
Speed at height 171.5 m: $v = 9.8 \times 1 = 9.8$ m s^{-1}
Speed of projection: $u^2 = v^2 + 2gh = 96.04 + 2 \times 9.8 \times 171.5 = 3457.44$
u = 58.8. Greatest height = $\frac{u^2}{(2g)} = \frac{3457.44}{19.6} = 176.4$

Answer: 58.8 m s^{-1}, 176.4 m

3 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cos(2\theta + 30°) = \frac{1}{2}$
for $0° \leq \theta \leq 180°$.

Let $u = 2\theta + 30°$; then $\cos u = \frac{1}{2}$
Base solutions: u = 60, −60° (+ multiples of 360°)
θ = (u − 30°) ÷ 2, keeping those in range
Answer: θ = 15°, 135°

4 Equations in disguise (substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 1\sqrt{x} - 6 = 0$.

Let $u = \sqrt{x}$: $u^2 - 1u - 6 = 0$.
(u − 3)(u + 2) = 0, so $\sqrt{x} = 3$ or $\sqrt{x} = -2$.
$\sqrt{x}$ cannot be negative, so reject $\sqrt{x} = -2$.
x = 9.
Answer: x = 9

5 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = \ln t$, $y = t^2 - 5t$, $t > 0$.
Find the equation of the tangent where t = 1, in the form y = mx + c.

$dx/dt = \frac{1}{t}$, $dy/dt = 2t - 5$
dy/dx = t(2t − 5) which is −3 at t = 1
At t = 1: x = 0, y = −4
Answer: y = −3x − 4

6 Cubic graph: find the equation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the cubic y = f(x), which touches the x-axis once. Find f(x), giving your answer in expanded form.

Crosses at $x = -3$ and touches at $x = -1$, so $f(x) = k(x + 3)(x + 1)^2$.

At (0, 3): $3 = k(3)(1)^2 = 3k$, so $k = 1$.

$f(x) = (x + 3)(x + 1)^2$, then expand.

Answer: $f(x) = x^3 + 5x^2 + 7x + 3$

7 Normal: unknown standard deviation then a second probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The lengths of a component, in mm are modelled by $N(43, \sigma^2)$. The probability of a value above 46 is 0.35.
(a) Find σ.
(b) Find the probability of a value above 40.

$P(X > 46) = 0.35$ so $Z = \frac{46 - 43}{\sigma} = 0.3853$

$\sigma = \frac{3}{0.3853} = 7.786$

$z = \frac{40 - 43}{7.786} = -0.385$

$P(X > 40) = P(Z > -0.385) = 0.650$

Answer: (a) 7.79 (b) 0.650

8 Volumes of revolution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region bounded by $y = \frac{1}{\sqrt{x + 3}}$, the x-axis and the lines
$x = 0$ and $x = 2$ is rotated through 2π about the x-axis.
Find the exact volume of the solid formed.

$V = \pi\int_0^2 y^2\,dx = \pi\int_0^2 \left(\frac{1}{\sqrt{x + 3}}\right)^2 dx$

Simplify the integrand, then integrate

$V = \pi \times (-\ln 3 + \ln 5)$

Answer: $-\pi\ln 3 + \pi\ln 5$

9 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|x - 5| < |2x - 1|$.

Square both sides: $(x - 5)^2 < (2x - 1)^2$.

$(-x - 4)(3x - 6) < 0$.

Critical values $x = -4$ and $x = 2$.

The quadratic has negative coefficient of x^2, so choose the outer region.

Answer: $x < -4$ or $x > 2$

10 Interception at constant speed (AQA, Edexcel, OCR, MEI, Eduqas)

At time t = 0 hours, ship A is at the point with position vector $(2\mathbf{i} + 6\mathbf{j})$ km relative to a port O, and moves with constant velocity $(2\mathbf{i} + \mathbf{j})$ km h^{-1}. Ship B leaves O at t = 0 with constant speed 5 km h^{-1} and intercepts A.
Find (a) the time at which B intercepts A, (b) the velocity of B as a vector.

Position of A at time t: $(2\mathbf{i} + 6\mathbf{j}) + t(2\mathbf{i} + \mathbf{j})$. B at time t: t v_B (unknown velocity)

At interception B has travelled 5 t, so $|(2\mathbf{i} + 6\mathbf{j}) + t(2\mathbf{i} + \mathbf{j})| = 5\,t$

Squaring and rearranging: $-20t^2 + 20t + 40 = 0$, which gives t = 2 or a negative value

t = 2 (the other root is negative since B is faster than A)

A is then at $(2\mathbf{i} + 6\mathbf{j}) + 2(2\mathbf{i} + \mathbf{j}) = (6\mathbf{i} + 8\mathbf{j})$, so $v_B = \frac{1}{2}(6\mathbf{i} + 8\mathbf{j}) = (3\mathbf{i} + 4\mathbf{j})$

Answer: (a) 2 h (b) $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1}