Day 314 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Partial fractions (repeated factor) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-3x^2 - 8x - 13}{(x + 3)(x - 1)^2}$ in the form
$\frac{A}{(x + 3)} + \frac{B}{(x - 1)} + \frac{C}{(x - 1)^2}$.

$-3x^2 - 8x - 13 \equiv A(x - 1)^2 + B(x + 3)(x - 1) + C(x + 3)$.
Put $x = -3$: $-16 = A(-4)^2$, so $A = -1$.
Put $x = 1$: $-24 = C(4)$, so $C = -6$.
Compare x^2: $-3 = 1A + 1B$, so $B = -2$.
Answer: A = −1, B = −2, C = −6

2 Distance travelled with a change of direction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on a straight line. At time t seconds its velocity is v m s^{-1}, where
$v = 2t^2 - 18t + 40$ for $0 \leq t \leq 7$
Find (a) the times at which P is at instantaneous rest, (b) the total distance travelled by P in the first 7 seconds.

$v = 2(t - 4)(t - 5)$, so $v = 0$ when $t = 4$ and $t = 5$
$v > 0$ for $0 < t < 4$, $v < 0$ for $4 < t < 5$, $v > 0$ for $t > 5$: direction changes at each

$s = \int v\,dt = \frac{2}{3}t^3 - 9t^2 + 40t$

Distances: 0 to 4: $\frac{176}{3}$; 4 to 5: $\frac{1}{3}$; 5 to 7: $\frac{28}{3}$

Total = $\frac{176}{3} + \frac{1}{3} + \frac{28}{3}$

Answer: (a) t = 4, t = 5 (b) $\frac{205}{3}$ m

3 Tree diagram with an unknown probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows two games played in turn.

Given the second is a W, P(first was a W) = $\frac{4}{13}$. Find x.

P(W then W) = 0.8x, P(second W) = 0.8x + 0.6(1 − x)

P(first W | second W) = $\frac{0.8x}{0.8x + 0.6(1 - x)} = \frac{4}{13}$

$0.8x = \frac{4}{13}(0.8x + 0.6 - 0.6x)$

Solve the linear equation for x: $x = \frac{1}{4}$

Answer: x = $\frac{1}{4}$

4 Equations quadratic in an exponential (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $e^{2x} - 9 \times e^x + 14 = 0$,
giving each answer exactly and, where not an integer, to 3 s.f.

Let $u = e^x$: $u^2 - 9u + 14 = 0$
$(u - 2)(u - 7) = 0$, so $e^x = 2$ or $e^x = 7$
Take logarithms: $x = \ln 2 = 0.693$; $x = \ln 7 = 1.95$
Answer: x = ln 2 = 0.693; x = ln 7 = 1.95

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5 Stratified sample then probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A school has 100 students in Year 10, 60 in Year 11 and 140 in the sixth form.
A stratified sample of 30 students is taken.
(a) Find the number sampled from each group.
(b) Two students are chosen at random from the sample. Find the probability they are from the same group.

Sample sizes: $30 \times \frac{100}{300}$, $30 \times \frac{60}{300}$, $30 \times \frac{140}{300}$ = 10, 6, 14

P(same) = $10 \times 9 + 6 \times 5 + 14 \times 13/30 \times 29$

$= \frac{302}{870} = 0.347$

Answer: (a) 10, 6, 14 (b) 0.347

6 Pair of inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which both
$x^2 - 6x - 7 \geq 0$ and $x + 4 > -6$.

Quadratic: $(x + 1)(x - 7) \geq 0$ gives $x \leq -1$ or $x \geq 7$.
Linear: $x + 4 > -6$ gives $x > -10$.
Both must hold: take the overlap of the two sets.
Answer: $-10 < x \leq -1$ or $x \geq 7$

7 Stationary points of x^n times a root (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2\sqrt{45 - x}$, for $0 < x < 45$, has one stationary point. Find its exact coordinates and determine its nature.

$\frac{dy}{dx} = \frac{x(180 - 5x)}{2\sqrt{45 - x}}$

dy/dx = 0: $180 - 5x = 0$, so $x = 36$

$y = 36^2 \times 3$

Gradient changes from positive to negative: a maximum
Answer: (36, 3888), maximum

8 Velocity with a square-root term (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P moves on a straight line. At time t seconds, where $t \geq 0$, its velocity is v m s^{-1}, where
$v = 12\sqrt{t} - 2t$.
Find (a) the greatest speed of P, (b) the greatest displacement of P from its starting point.

$\frac{dv}{dt} = \frac{12}{2\sqrt{t}} - 2 = 0$ when $\sqrt{t} = 3$, so $t = 9$

$v = 12 \times 3 - 2 \times 9 = 18$

v = 0 when $\sqrt{t}(12 - 2\sqrt{t}) = 0$: $t = 36$ ($t = 0$ is the start)

$s = \int v\,dt = \frac{2}{3} \times 12\, t^{\frac{3}{2}} - \frac{2}{2}t^2$, so at $t = 36$: $s = 1728 - 1296$

Answer: (a) 18 m s^{-1} (b) 432 m

9 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|4x + 6| \leq |2x - 4|$.

Square both sides: $(4x + 6)^2 \leq (2x - 4)^2$.
$(2x + 10)(6x + 2) \leq 0$.

Critical values $x = -5$ and $x = -\frac{1}{3}$.

The quadratic has positive coefficient of x^2, so choose the inner region.

Answer: $-5 \leq x \leq -\frac{1}{3}$

10 Closest approach of moving points (Edexcel, OCR, MEI, Eduqas, CIE (vectors in kinematics))

Two boats P and Q move so that, t hours after noon, their position vectors in km are

$\mathbf{r}_P = \begin{pmatrix} -5 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{r}_Q = \begin{pmatrix} -7 \\ 6 \end{pmatrix} + t\begin{pmatrix} 5 \\ 1 \end{pmatrix}$.

Find the time at which they are closest and the exact minimum distance between them.

$\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + t\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Distance$^2 = |\mathbf{r}_Q - \mathbf{r}_P|^2$, minimised when $(\mathbf{r}_Q - \mathbf{r}_P) \cdot \begin{pmatrix} 2 \\ 0 \end{pmatrix} = 0$

$-4 + t(4) = 0$, so $t = 1$

Then $\mathbf{r}_Q - \mathbf{r}_P = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$, with length $\sqrt{4}$

Answer: t = 1, minimum distance = 2 km