Day 308 **A Level Maths: S Tier 10-a-day: Model Solutions**

1 Trig equations with compound angles (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\cos(2\theta - 30°) = -\frac{1}{2}$
for $0° \leq \theta \leq 180°$.

Let $u = 2\theta - 30°$; then $\cos u = -\frac{1}{2}$
Base solutions: $u = 120, -120°$ (+ multiples of 360°)
$\theta = (u + 30°) \div 2$, keeping those in range
Answer: $\theta = 75°, 135°$

2 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=5}^{10}(2 \times 2^r + 4r)$.

Geometric part: first term $2 \times 2^5 = 64$, ratio 2, 6 terms.
Sum = $\frac{64(2^6 - 1)}{1} = 4032$.
Arithmetic part: 4×6 terms, average $(5 + 10)/2$: sum = 180.
Total = 4032 + 180.
Answer: 4212

3 Rough table and smooth slope: string cut (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (1 kg) lies on a rough table with $\mu = 0.3$, and is joined by a light string over a smooth pulley (see diagram) to particle B (10 kg) on a smooth slope, $\sin\alpha = \frac{4}{5}$.
Released from rest, the string is cut when B has moved 3.75 m. Find the further distance A slides before stopping.

A: $T - 0.3 \times 1 \times 9.8 = 1\,a$. B: $10 \times 9.8 \times \frac{4}{5} - T = 10\,a$
Adding: $9.8(10 \times \frac{4}{5} - 0.3 \times 1) = 11\,a$, so $a = 6.86$
$T = 1(6.86 + 0.3 \times 9.8) = 9.8$
When cut: $v^2 = 2 \times 6.86 \times 3.75 = 51.45$. Then A decelerates at $\mu g = 2.94$
Further distance = $\frac{51.45}{2 \times 2.94} = 8.75$
Answer: 8.75 m

4 Arithmetic terms forming a geometric sequence (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic sequence is 2 and its common difference d is non-zero. The 4th, 6th and 9th terms are the first three terms of a geometric sequence. Find d and the common ratio.

Terms: $2 + 3d$, $2 + 5d$, $2 + 8d$.
Geometric: $(2 + 5d)^2 = (2 + 3d)(2 + 8d)$.
Expand: $25d^2 + 20d + 4 = 24d^2 + 22d + 4$, so $d(d - 2) = 0$.
$d \neq 0$, so $d = 2$. Ratio = $\frac{12}{8} = \frac{3}{2}$.
Answer: $d = 2$, ratio = $\frac{3}{2}$

5 Resultant of forces: unknown constant and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-8\mathbf{i} - 7\mathbf{j})$ N, $(-1\mathbf{i} + k\mathbf{j})$ N and $(-8\mathbf{i} - 4\mathbf{j})$ N act on a particle of mass 2 kg. The resultant is parallel to $(-17\mathbf{i} - 14\mathbf{j})$.
Find (a) the value of k, (b) the magnitude of the acceleration of the particle, to 3 s.f.

Resultant = $(-17)\mathbf{i} + (-11 + k)\mathbf{j}$
Parallel to $(-17\mathbf{i} - 14\mathbf{j})$: $\frac{-11 + k}{-17} = \frac{-14}{-17}$, so $k = -3$
$R = (-17\mathbf{i} - 14\mathbf{j})$ N, so $|R| = \sqrt{(-17^2 + -14^2)} = 22.02$
$a = |R| \div 2$
Answer: (a) $k = -3$ (b) 11.0 m s^{-2}

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6 Integration by substitution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution u = x + 1 to find the exact value of

$\int_0^{35} \frac{x}{\sqrt{x+1}}\,dx$

x = u − 1, dx = du; limits become u = 1 and u = 36

Integral = $\int_1^{36} \frac{u-1}{\sqrt{u}}\,du = [\frac{2}{3}u^{3/2} - 2u^{1/2}]$

Evaluate the limits

Answer: $\frac{400}{3}$

7 Continuous random variable: pdf with unknown constant (CIE)

The pdf of X is $f(x) = \frac{k}{x^2}$ for $1 \le x \le 7$, and 0 otherwise.

(a) Find k.
(b) Find the median of X.
(c) Find E(X), to 3 significant figures.

(a) $\int k\,x^{-2}\,dx$ from 1 to 7 = $k(1 - \frac{1}{7}) = 1$, so $k = \frac{7}{6}$

(b) $\int f(x)\,dx$ from 1 to m = $\frac{7}{6}(1 - \frac{1}{m}) = \frac{1}{2}$

$1 - \frac{1}{m} = \frac{3}{7}$, so $m = \frac{7}{4}$

(c) $E(X) = \int \frac{7}{6}x^{-1}\,dx = \frac{7}{6}\ln 7 = 2.27$

Answer: (a) k = $\frac{7}{6}$ (b) $\frac{7}{4}$ (c) 2.27

8 Implicit curves: horizontal and vertical tangents (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $x^2 - xy + 3y^2 = 99$.
Find the coordinates of the points on C where the tangent is parallel to the x-axis.

Differentiate: $2x + by + (bx + 2cy)dy/dx = 0$, so $dy/dx = -\frac{2x + -y}{-x + 6y}$

Horizontal tangent when $2x + -y = 0$, so $y = \frac{2x}{1}$

Substitute into the equation of C: $x^2 + (-xy) + (3y^2) = 99$
Solve for the remaining coordinate
Answer: (−3, −6) and (3, 6)

9 Independence from a Venn diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The Venn diagram shows probabilities; A and B are independent.
Find x and y.

P(A) = 2x + x = 3x, P(B) = x + y, $P(A \cap B) = x$

Independent: $x = 3x(x + y)$, and $x \neq 0$, so $x + y = \frac{1}{3}$

Total probability: $2x + x + y + 0.2 = 1$, so $2x + (x + y) = 0.8$

$2x = 0.8 - \frac{1}{3} = \frac{7}{15}$

$x = \frac{7}{30}$, $y = \frac{1}{3} - x = \frac{1}{10}$

Answer: x = $\frac{7}{30}$, y = $\frac{1}{10}$

10 Parametric curve and line intersection (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has parametric equations

$x = 3t$, $y = 2t^2$. The line $y = -\frac{4}{3}x + 6$

meets C at A and B. Find the exact length AB.

Substitute: $2t^2 = -4t + 6$.

$2t^2 + 4t - 6 = 0 \Rightarrow t = 1$ or $t = -3$.

A(3, 2), B(−9, 18).

$AB^2 = 12^2 + (-16)^2 = 400$.

Answer: AB = 20